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MONITORING OF SLIDING CONTACT WITH WEAR BY MEANS OF PIEZOELECTRIC INTERLAYER PARAMETERS

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Abstract. The contact problem on frictional sliding of a rigid body over a coating's surface is considered. During sliding, frictional heating and wear of the coating takes place at the contact interface. A piezoelectric interlayer is placed between the coating and the rigid substrate, which edges are perfectly bonded to the substrate and to the lower boundary of the coating. Electrodes are located at the edges of the interlayer, being connected to the control circuit and subjected to electric potential difference. Solutions of the problem were represented in form of the Laplace convolutions. They allow to determine relationship between electric current in the interlayer and main parameters of the contact: temperature, contact stresses, displacements, coating's wear. Also the obtained solutions show that one can alter contact parameters by changing the potential difference on electrodes of the interlayer.

Keywords: wear, sliding contact, thermoelasticity, piezoelectricity, coating, piezoelectric interlayer

1. Introduction

Operation of high-speed vehicles, industrial equipment, etc. is accompanied by an increase in loads in the frictional joints of machines and mechanisms, accelerated wear of working surfaces, their heating, the emergence of critical situations. The problem of creating frictional surfaces that meet the increased operational requirements is often solved by the use of coatings for various purposes: anti-friction, anti-corrosion, thermal insulation, etc. It has been experimentally established that an increase in the relative velocity between the working surfaces of tribotechnical devices with coatings generates a rapid increase in temperature and contact stress, indicating the development of thermoelastic instability of the sliding frictional contact [1–8].

Piezoelectric sensors are widely used for monitoring the parameters of the sliding frictional contact. Mechanics of indentation of piezoelectric material was considered in [9-11]. However, the fragility and thermal sensitivity of piezoceramics does not allow placing piezoelectric sensors near the contact. Thermoelastic problems on a rigid body frictional sliding over the surface of an elastic coating with piezoelectric interlayer but without wear were considered previously in [12,13].

In the present work, in order to study the possibilities of indirect monitoring of the level of coating wear, contact stresses and temperature, a transient thermoelastic/electroelastic problem is considered on the sliding contact of a rigid plate over a surface of an elastic

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coating equipped with a heat-insulated piezoelectric layer that allows monitoring the main parameters of contact and wear of the coating.

2. Problem statement

The rigid half-plane *I* slides with constant speed *V* over the surface of the elastic coating *A* with thickness *h*, bonded by its lower boundary to the electroelastic thermally insulated interlayer *B* with thickness *H* (Fig. 1). Polarization vector of the piezoelectric material is normal to the boundaries of the interlayer. By its lower boundary the interlayer is bonded to the rigid substrate in the form of the half-plane *II*. During sliding, the half-plane *I* penetrates the coating by normal to its surface. Sliding of the thermally insulated rigid half-plane *I* is performed with account for Coulomb friction and wear of the coating. Thermal flux generated by friction on the contact interface is directed to the coating *A*. Boundaries of the electroelastic interlayer *B* are covered with electrodes with applied potential difference. At the initial moment, displacements and their velocities in the coating and the interlayer are zero, and the initial temperature of the coating is also zero.

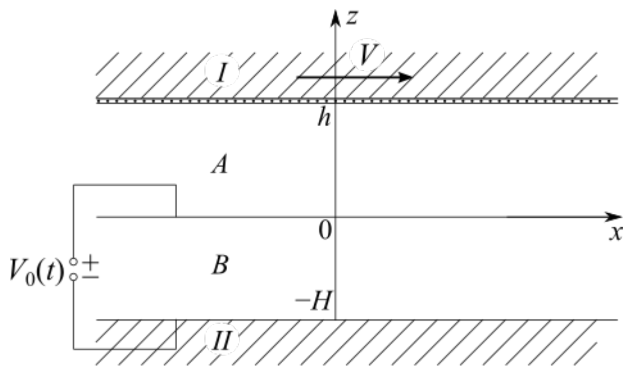

Fig. 1. Contact geometry

According to the given formulation, all main physical parameters of the problem, namely temperature, stresses, displacements, electric induction and electric intensity, do not depend on the horizontal coordinate. In this case, behavior of the thermoelastic coating *A* is described by system of differential equation of thermoelasticity together with heat equation [14] at zero initial conditions

$$\frac{\partial^2 u}{\partial z^2} = \frac{1+\nu}{1-\nu}\alpha\frac{\partial T}{\partial z}, \quad \frac{\partial^2 w}{\partial z^2} = 0, \quad \frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa}\frac{\partial T}{\partial t} = 0, \quad 0 < z < h, \quad t > 0, \tag{1}$$

where $u(z,t)$, $w(z,t)$ are the vertical and horizontal displacements, $T(z,t)$ is the temperature, μ, ν, α, κ are, respectively, the shear modulus, Poisson's ratio, linear thermal expansion coefficient and thermal diffusivity of the coating *A* material. The Duhamel – Neumann relationships define the connection between stresses, displacements and temperature

$$\sigma_{zz} = \frac{2\mu(1-\nu)}{1-2\nu}\frac{\partial u}{\partial z} - \frac{2\mu(1+\nu)}{1-2\nu}\alpha T, \quad \sigma_{xz} = \mu\frac{\partial w}{\partial z}, \tag{2}$$

where $\sigma_{zz}(z,t)$, $\sigma_{xz}(z,t)$ are the normal and tangential stresses in the coating.

Behavior of the electroelastic interlayer *B* is described by system of differential equations of electroelasticity of a piezoceramic material polarized in the direction of z axis [15] at zero initial conditions

$$\frac{\partial^2 u_1}{\partial z^2} = 0, \quad \frac{\partial^2 w_1}{\partial z^2} = 0, \quad \frac{\partial^2 \psi}{\partial z^2} = 0, \quad -H < z < 0, \quad t > 0, \tag{3}$$

where $u_1(z,t)$, $w_1(z,t)$, $\psi(z,t)$ are, respectively, the vertical and horizontal displacement and the electric potential inside the interlayer *B*. Mechanical stresses and electric intensity in the piezoceramic interlayer are taken in form

$$\sigma_{zz}^1 = c_{33}^E \frac{\partial u_1}{\partial z} + e_{33}\frac{\partial \psi}{\partial z}, \quad \sigma_{xz}^1 = c_{44}^E \frac{\partial w_1}{\partial z}, \quad D_z = -\varepsilon_{33}^S \frac{\partial \psi}{\partial z} + e_{33}\frac{\partial u_1}{\partial z}, \tag{4}$$

where $\sigma_{zz}^1(z,t)$, $\sigma_{xz}^1(z,t)$ are normal and tangential stresses in the interlayer, c_{33}^E, c_{44}^E are the elastic moduli measured at constant electric field, ε_{33}^S is the dielectric permittivity measured at constant deformation, e_{33} is the piezoelectric modulus of the interlayer *B*..

Mechanical, temperature and electric boundary conditions of the formulated quasi-static problem on sliding contact are written as follows:

$$z = h \qquad u(h,t) = -\Delta(t) + u_w(t), \quad \sigma_{xz}(h,t) = -f\sigma_{zz}(h,t), \quad K\frac{\partial T(h,t)}{\partial z} = -fV\sigma_{zz}(h,t); \tag{5}$$

$$z = 0 \qquad u(0,t) = u_1(0,t), \quad w(0,t) = w_1(0,t), \quad T(0,t) = 0; \tag{6}$$

$$\sigma_{zz}(0,t) = \sigma_{zz}^1(0,t), \quad \sigma_{xz}(0,t) = \sigma_{xz}^1(0,t), \quad \psi(0,t) = V_0(t);$$

$$z = -H \quad u_1(-H,t) = 0, \quad w_1(-H,t) = 0, \quad \psi(-H,t) = -V_0(t), \tag{7}$$

where f is the friction coefficient, V is the sliding velocity, K is the thermal conductivity of the coating *A* material, $\Delta(t)$ is the depth of the half-plane *I* indentation into the elastic coating, $2V_0(t)$ is the potential difference applied to the interlayer *B* electrodes.

The Archard's relationship [5-7] is taken as the wear model in (5):

$$u_w(t) = -fVK^*\int_0^t \sigma_{xx}(h,\tau)d\tau, \quad t > 0, \tag{8}$$

where K^* is the coefficient between the work of frictional forces and the amount of removed material.

The electric current I_0 through the piezoelectric interlayer *B*, divided by the cros-section area, is determined from the equation

$$I_0 = \frac{\partial D_z}{\partial t}, \quad t > 0. \tag{9}$$

Horizontal displacements $w(z,t)$ are determined from vertical displacements $u(z,t)$ when latter are known.

3. Exact solution of the problem

By using the Laplace integral transform [16] the solution of quasi-static initial boundary value problem on the sliding contact (1)–(9) was written in the form of the Laplace convolutions containing the vertical displacement $\Delta(t)$ of the sliding half-plane *I* and the potential difference $V_0(t)$ on the piezoelectric interlayer *B* electrodes:

$$T(z,t) = \frac{1-\nu}{1+\nu}\frac{\hat{V}}{\alpha h}\left(\int_0^t \Delta(\tau) f_T^0(z,t-\tau)d\tau - \theta\int_0^t V_0(\tau) g_T^0(z,t-\tau)d\tau\right), \ 0 \le z \le h; \tag{10}$$

$$u(z,t) = -\int_0^t \Delta(\tau) f_u^0(z,t-\tau)d\tau + \theta\int_0^t V_0(\tau) g_u^0(z,t-\tau)d\tau, \ 0 \le z \le h; \tag{11}$$

$$\sigma_{zz}(z,t) = -\frac{2\mu(1-\nu)}{(1-2\nu)h}\left(\int_0^t \Delta(\tau) f_\sigma^0(z,t-\tau)d\tau - \theta\int_0^t V_0(\tau) g_\sigma^0(z,t-\tau)d\tau\right),\ 0 \le z \le h; \tag{12}$$

$$w(z,t) = -\frac{f}{\mu}\left(z + H\frac{\mu}{c_{44}^E}\right)\sigma_{zz}(z,t),\ 0 \le z \le h; \tag{13}$$

$$u_1(z,t) = -\int_0^t \Delta(\tau) f_{u_1}^0(z,t-\tau)d\tau + \theta\int_0^t V_0(\tau) g_{u_1}^0(z,t-\tau)d\tau,\ -H \le z \le 0; \tag{14}$$

$$\sigma_{1zz}(z,t) = -\frac{2\mu(1-\nu)}{(1-2\nu)h}\left(\int_0^t \Delta(\tau) f_\sigma^0(z,t-\tau)d\tau - \theta\int_0^t V_0(\tau) g_\sigma^0(z,t-\tau)d\tau\right),\ -H \le z \le 0; \tag{15}$$

$$w_1(z,t) = -f\mu(c_{44}^E)^{-1}(z+H)\sigma_{zz}(z,t),\ -H \le z \le 0; \tag{16}$$

$$\psi(z,t) = (2z+H)H^{-1}V_0(t),\ -H \le z \le 0; \tag{17}$$

$$I_0(t) = -\frac{e_{33}}{Ht_\kappa}\left(\int_0^t \Delta(\tau) f_I^0(0,t-\tau)d\tau - \theta\int_0^t V_0(\tau) g_I^0(0,t-\tau)d\tau\right) - \frac{\varepsilon_{33}^S}{Ht_\kappa}\cdot 2V_0^L(t), \tag{18}$$

where

$$f_a^0(z,t) = \frac{1}{2\pi i}\int_\Gamma \frac{N_a^0(z,\zeta)}{t_\kappa R(\zeta)} e^{\zeta\tilde{t}} d\zeta,\quad g_a^0(z,t) = \frac{1}{2\pi i}\int_\Gamma \frac{M_a^0(z,\zeta)}{t_\kappa R(\zeta)} e^{\zeta\tilde{t}} d\zeta, \tag{19}$$

$\tilde{t} = t/t_\kappa$, $\Gamma = \{\zeta : -i\infty + dt_\kappa, i\infty + dt_\kappa\}$ is the contour of integration, where d is chosen in a way that all isolated singularities of the integrands in (19) would be placed left to the contour Γ. The functions $N_a^0(z,\zeta)$, $M_a^0(z,\zeta)$ and $R(z)$ have the form

$$\begin{aligned}
&N_T^0(z,\zeta) = M_T^0(z,\zeta) = \sqrt{\zeta}\,\mathrm{sh}(\sqrt{\zeta} z h^{-1}) && 0 \le z \le h\\
&N_u^0(z,\zeta) = (zh^{-1} + \eta)\zeta\,\mathrm{ch}\sqrt{\zeta} - \hat{V}\left(\mathrm{ch}\sqrt{\zeta} z h^{-1} - 1\right) && 0 \le z \le h\\
&M_u^0(z,\zeta) = (z-h)h^{-1}\zeta\,\mathrm{ch}\sqrt{\zeta} - \hat{V}\left(\mathrm{ch}\sqrt{\zeta} z h^{-1} - (1-k_\mathrm{w})\,\mathrm{ch}\sqrt{\zeta}\right) && 0 \le z \le h\\
&N_\sigma^0(z,\zeta) = M_\sigma^0(z,\zeta) = \zeta\,\mathrm{ch}\sqrt{\zeta} && 0 \le z \le h\\
&N_{u_1}^0(z,\zeta) = (zH^{-1} + 1)\eta\zeta\,\mathrm{ch}\sqrt{\zeta} && -H \le z \le 0\\
&M_{u_1}^0(z,\zeta) = -(zH^{-1} + 1)\left(\zeta\,\mathrm{ch}\sqrt{\zeta} - \hat{V}((1-k_\mathrm{w})\,\mathrm{ch}\sqrt{\zeta} - 1)\right) && -H \le z \le 0\\
&N_{\sigma_1}^0(z,\zeta) = M_{\sigma_1}^0(z,\zeta) = \zeta\,\mathrm{ch}\sqrt{\zeta} && -H \le z \le 0\\
&N_I^0(z,\zeta) = \eta\zeta^2\,\mathrm{ch}\sqrt{\zeta} && -H \le z \le 0\\
&M_I^0(z,\zeta) = -\zeta\left(\zeta\,\mathrm{ch}\sqrt{\zeta} - \hat{V}((1-k_\mathrm{w})\,\mathrm{ch}\sqrt{\zeta} - 1)\right) && -H \le z \le 0\\
&R(\zeta) = (1+\eta)\zeta\,\mathrm{ch}\sqrt{\zeta} - \hat{V}((1-k_\mathrm{w})\,\mathrm{ch}\sqrt{\zeta} - 1).
\end{aligned} \tag{20}$$

In (10)–(20), the following notation is used

$$t_\kappa = \frac{h^2}{\kappa},\ \eta = \frac{H_*}{\delta_0},\ \delta_0 = c_{33}^E \Big/ \frac{2\mu(1-\nu)}{1-2\nu},\ \theta = \frac{2e_{33}}{c_{33}^E},\ H_* = \frac{H}{h},$$
$$\hat{V} = \frac{fV\alpha}{K}\frac{2\mu(1+v)h}{1-2v},\ k_\mathrm{w} = \frac{1-v}{1+v}\frac{KK^*}{\alpha\kappa}. \tag{21}$$

The analysis shown that some of the obtained quadratures exist only in generalized sense [17]. After isolating the generalized component of the formula, the solutions of the problem taken the following form:

$$T(z,t)=\frac{1-\nu}{1+\nu}\frac{\hat{V}}{\alpha h}\left[\frac{1}{1+\eta}\int_0^t \Delta_V(\tau)\gamma_T(z,t-\tau)d\tau+\int_0^t \Delta_V(\tau) f_T(z,t-\tau)d\tau\right] \qquad 0\le z\le h \qquad (22)$$

$$\gamma_T(z,t)=-\sqrt{\frac{t_\kappa}{t}}q_0(z,t)-2\sqrt{\frac{t_\kappa}{t}}\sum_{n=1}^{\infty}(-1)^n q_n(z,t)\,\mathrm{ch}\left(\frac{h-z}{h}\frac{t_\kappa}{t}n\right)$$

$$q_n(z,t)=\exp\left[-\frac{t_\kappa}{t}\left(\left(\frac{z-h}{2h}\right)^2+n^2\right)\right]$$

$$u(z,t)=-\frac{\eta+zh^{-1}}{\eta+1}\Delta(t)-\int_0^t \Delta(\tau) f_u(z,t-\tau)d\tau+\frac{\theta}{1+\eta}\int_0^t V_0(\tau)\beta_u(z,t-\tau)d\tau$$
$$+\theta\int_0^t V_0(\tau)g_u(z,t-\tau)d\tau \qquad 0\le z\le h \qquad (23)$$

$$\beta_u(z,t)=-\frac{1}{\sqrt{\pi t_\kappa t}}\left(\frac{z-2h}{2h}r_0(z,t)+\sum_{n=1}^{\infty}(-1)^n r_n(z,t)\psi_n(z,t)\right)$$

$$r_n(z,t)=\exp\left[-\frac{t_\kappa}{t}\left(\left(\frac{z-2h}{2h}\right)^2+n^2\right)\right]$$

$$\psi_n(z,t)=\frac{z-2h}{h}\mathrm{ch}\left(\frac{z-2h}{h}\frac{t_\kappa}{t}n\right)-2n\,\mathrm{sh}\left(\frac{z-2h}{h}\frac{t_\kappa}{t}n\right)$$

$$\sigma_{zz}(z,t)=-\frac{2\mu(1-\nu)}{(1-2\nu)h}\left[\frac{1}{1+\eta}\Delta_V(t)+\int_0^t \Delta_V(\tau) f_\sigma(z,t-\tau)d\tau\right] \qquad 0\le z\le h \qquad (24)$$

$$u_1(z,t)=-\frac{zH^{-1}+1}{\eta+1}(\eta\Delta(t)-\theta V_0(t))-\int_0^t \Delta(\tau) f_{u_1}(z,t-\tau)d\tau+$$
$$\theta\int_0^t V_0(\tau)g_{u_1}(z,t-\tau)d\tau \qquad -H\le z\le 0 \qquad (25)$$

$$\sigma_{1zz}(z,t)=-\frac{2\mu(1-\nu)}{(1-2\nu)h}\left[\frac{1}{1+\eta}\Delta_V(t)+\int_0^t \Delta_V(\tau) f_{\sigma_1}(z,t-\tau)d\tau\right] \qquad -H\le z\le 0 \qquad (26)$$

$$I_0(t)=-\frac{e_{33}}{Ht_\kappa}\left[\frac{t_\kappa}{1+\eta}\dot{\Delta}_V(t)+\frac{\eta\hat{V}(1-k_w)}{(1+\eta)^2}\Delta_V(t)+\int_0^t \Delta(\tau) f_I(0,t-\tau)d\tau+\right.$$
$$\left.\theta\int_0^t V_0(\tau)g_I(0,t-\tau)d\tau\right] \qquad (27)$$

where

$$f_a(z,t)=\frac{1}{2\pi i}\int_\Gamma \frac{N_a(z,\zeta)}{t_\kappa R(\zeta)}e^{\zeta\tilde{t}}d\zeta,\quad g_a(z,t)=\frac{1}{2\pi i}\int_\Gamma \frac{N_a(z,\zeta)}{t_\kappa R(\zeta)}e^{\zeta\tilde{t}}d\zeta \qquad (28)$$

$$N_a(z,\zeta)=N_a^0(z,\zeta)-\gamma_a^0(z,\zeta)R(\zeta),\quad M_a(z,\zeta)=M_a^0(z,\zeta)-\beta_a^0(z,\zeta)R(\zeta)$$

$$\gamma_T^0(z,\zeta)=\frac{1}{1+\eta}\frac{\mathrm{sh}\sqrt{\zeta}zh^{-1}}{\sqrt{\zeta}\,\mathrm{ch}\sqrt{\zeta}},\quad \gamma_u^0(z,\zeta)=\frac{zh^{-1}+\eta}{1+\eta},\quad \beta_u^0(z,\zeta)=\frac{zh^{-1}-1}{\eta+1}$$

$$\gamma_\sigma^0(z,\zeta)=\gamma_{\sigma_1}^0(z,\zeta)=\frac{1}{1+\eta},\quad \gamma_{u_1}^0(z,\zeta)=-\eta\beta_{u_1}^0(z,\zeta)=\frac{1+zH^{-1}}{1+\eta}\eta$$

$$\gamma_I^0(z,\zeta)=\frac{\eta}{1+\eta}\zeta+\frac{\eta}{(1+\eta)^2}\hat{V}(1-k_{\text{w}}),\quad \beta_I^0(z,\zeta)=-\frac{1}{1+\eta}\zeta+\frac{\eta}{(1+\eta)^2}\hat{V}(1-k_{\text{w}})$$

$\Delta_V(t)=\Delta(t)-\theta V_0(t)$, and $\dot{\Delta}_V(t)$ in (27) is given by $\dot{\Delta}_V(t)=\eta\dot{\Delta}(t)-\theta\dot{V}_0(t)$.

Formulas (22)–(27) made it possible to effectively investigate behavior of the obtained solution at small t and determine effect which the problem parameters have on contact temperature and stresses, and also on the electric current in the interlayer. As it can be seen from (21), decrease in elastic modulus or increase in thickness of the piezoelectric interlayer will lead to increase of the parameter η. According to (22), it will lead to a decrease in the contact temperature. From (23) it can be seen that increase of applied potential difference $V_0(t)$ will lead to the elastic expansion or shrinkage of the piezoelectric interlayer, depending on the sign of $V_0(t)$. Formula (27) indicates that the electric current in the interlayer is sensitive to the indentation rate $\dot{\Delta}(t)$ and to the applied potential difference rate $\dot{V}_0(t)$. Further, the influence of $\Delta(t)$ and $V_0(t)$ depends on magnitude of dimensionless parameter k_{w}, connected to the coating wear $u_w(t)$, and dimensionless parameter $\hat{V}$, which itself is proportional to friction coefficient *f*, sliding velocity *V*, thermal expansion coefficient α and elastic modulus μ of the coating material and other parameters of the problem.

Note that contour quadratures for $\sigma_{zz},\sigma_{zz}^1,I_0$ do not depend on the coordinate z. Moreover, the stresses σ_{zz} in the coating *A* and σ_{zz}^1 in the interlayer *B* coincide.

The investigation of integrands in (28) shows that they are all meromorphic in the complex plane of the integration variable $\zeta=\xi+i\eta$ and have only poles as their isolated singularities. To effectively calculate the contour quadratures, it is necessary to investigate and determine poles of the integrands. The calculation of contour quadratures in (28) using the residue theorem, similarly to [7,8,12,13], allows one to construct an effective solution for any $t\in(0,\infty)$.

4. Effective solution of the problem

For the contour integrals (28) to exist, the integrands should decay by the power law at infinity $|\zeta|\to\infty$. Analysis of the integrands (28) gave the following asymptotic estimations at $|\zeta|\to\infty$

$$N_T R^{-1}=M_T R^{-1}=\mathrm{O}(\zeta^{-1/2})$$

$$\{N_u,M_u,N_\sigma,N_{u_1},M_{u_1},N_{\sigma_1},N_I,M_I\}R^{-1}=\mathrm{O}(\zeta^{-1}), \tag{29}$$

where

$$N_a=N_a(z,\zeta),\ M_a=M_a(z,\zeta),\quad a=T,u,\sigma,u_1,\sigma_1,I,\quad R=R(\zeta).$$

Let us take into account the integrands behavior at infinity (29), their meromorphity in the complex plane and consider all of their poles ζ_k $k=0,1,2,...$ to be simple. Then, to effectively calculate integrals (28), the complex analysis methods [18] can be used, giving the formulas

$$f_a(z,t)=\sum_{k=0}^{\infty}B_a^N(z,\zeta_k)e^{\zeta_k\tilde{t}},\ g_a(z,t)=\sum_{k=0}^{\infty}B_a^M(z,\zeta_k)e^{\zeta_k\tilde{t}} \tag{30}$$

$$B_a^N(z,\zeta)=\frac{N_a(z,\zeta)}{t_\kappa R'(\zeta)},\ B_a^M(z,\zeta)=\frac{M_a(z,\zeta)}{t_\kappa R'(\zeta)},$$

where the integrand poles ζ_k are sorted by their absolute values: $|\zeta_0|\le|\zeta_1|\le...\le|\zeta_k|\le...$ The index a takes one of the symbols: $T,u,\sigma,u_1,\sigma_1,I$.

By substituting (30) to (22)–(28) the problem solution was obtained in the form of series over the poles ζ_k $k = 0,1,2,...$, effective at $t > 0$

$$T(z,t) = \frac{1-\nu}{1+\nu}\frac{\hat{V}}{\alpha h}\sum_{k=0}^{\infty}\left[\frac{1}{1+\eta}\int_0^t \Delta_V(\tau)\gamma_T(z,t-\tau)d\tau + \sum_{k=0}^{\infty} B_T^N(z,\zeta_k)D_{\Delta_V}(\zeta_k,t)\right] \quad 0 \le z \le h \quad (31)$$

$$u(z,t) = -\frac{\eta + zh^{-1}}{\eta+1}\Delta(t) + \frac{\theta}{1+\eta}\int_0^t V_0(\tau)\beta_u(z,t-\tau)d\tau - \sum_{k=0}^{\infty}\left(B_u^N(z,\zeta_k)D_\Delta(\zeta_k,t) - \theta B_u^M(z,\zeta_k)D_V(\zeta_k,t)\right) \quad 0 \le z \le h \quad (32)$$

$$\sigma_{zz}(z,t) = -\frac{2\mu(1-\nu)}{(1-2\nu)h}\left[\frac{1}{1+\eta}\Delta_V(t) + \sum_{k=0}^{\infty} B_\sigma^N(z,\zeta_k)D_{\Delta_V}(\zeta_k,t)\right] \quad 0 \le z \le h \quad (33)$$

$$u_1(z,t) = -\frac{1+zH^{-1}}{1+\eta}\left(\eta\Delta(t) - \theta V_0(t)\right) - \sum_{k=0}^{\infty}\left(B_{u_1}^N(z,\zeta_k)D_\Delta(\zeta_k,t) - \theta B_{u_1}^M(z,\zeta_k)D_V(\zeta_k,t)\right) \quad -H \le z \le 0 \quad (34)$$

$$\sigma_{1zz}(z,t) = -\frac{2\mu(1-\nu)}{(1-2\nu)h}\left[\frac{1}{1+\eta}\Delta_V(t) + \sum_{k=0}^{\infty} B_{\sigma_1}^N(z,\zeta_k)D_{\Delta_V}(\zeta_k,t)\right] \quad -H \le z \le 0 \quad (35)$$

$$I_0(t) = -\frac{e_{33}}{Ht_\kappa}\left[\frac{t_\kappa}{1+\eta}\dot{\Delta}_V(t) + \frac{\eta\hat{V}}{(1+\eta)^2}\Delta_V(t) + \sum_{k=0}^{\infty}\left(B_I^N(z,\zeta_k)D_\Delta(\zeta_k,t) - \theta B_I^M(z,\zeta_k)D_V(\zeta_k,t)\right)\right] \quad -H \le z \le 0 \quad (36)$$

The horizontal displacements $w(z,t), w_1(z,t)$ and the electric potential $\psi(z,t)$ are given, respectively, by equations (13), (16) and (17). The tangential stresses $\sigma_{xz}(z,t)$ and the coating wear $u_w(t)$ are determined by the boundary condition (5). The formulas (31)–(36) use functions from (22)–(27) and (30) and the following notation

$$D_\Delta(\zeta,t) = \int_0^t \Delta(\tau)\exp\left(\frac{\zeta(t-\tau)}{t_\kappa}\right)d\tau, \quad D_V(\zeta,t) = \int_0^t V_0(\tau)\exp\left(\frac{\zeta(t-\tau)}{t_\kappa}\right)d\tau,$$

$$D_{\Delta V}(\zeta,t) = D_\Delta(\zeta,t) - \theta D_V(\zeta,t).$$

5. Numerical analysis of the obtained solution

The solution of the thermoelastic quasi-static problem on coating wear was obtained above. Here, the numerical analysis of the obtained solution is conducted for the temperature $T(x,t)$, wear $u_w(t)$ (5), (32), mechanical stress $\sigma_{zz}(x,t)$ (33) and electric current through the piezoelectric interlayer $I_0(t)$ (36). Let $\Delta_0 = 0{,}01h$ be the maximum penetration of the elastic coating by the rigid half-plane *I* be, and the indentation law $\Delta(t)$ have the form

$$\Delta(t) = \Delta_0 H(t), \quad (37)$$

where $H(t)$ is the Heaviside function. Let us also consider the potential difference $V_0(t)$ between the piezoelectric interlayer electrodes to be equal zero. Then the mechanical interaction will be only responsible for the current through the piezoelectric interlayer.

Let us consider effect of the sliding velocity V on the main contact parameters: the temperature $T(h,t)$ (17), the contact stress $p(t) = -\sigma_{xx}(h,t)$ from (19), the coating wear $u_w(t) = \Delta(t) + u(h,t)$ from (18) and the electric current $I_0(t)$ from (22). The elastic coating is made of the aluminum alloy with the following thermomechanical properties: μ = 36.0 GPa, ν = 0.35, κ = 33.5 · 10^{-6} m^2/s, α = 18.4 · 10^{-6} 1/K, K = 111 W/(m·K), h = 0.5 mm, f = 0.15, $K^* = 7.5 \cdot 10^{-12}$ m^2/N. The piezoelectric interlayer is made of the PZT-4 piezoceramics with the following electromechanical properties: H = 0.01 mm, c_{33}^E = 115 GPa, e_{33} = 15.1 C/m^2, $\varepsilon_{33}^S = 635\varepsilon_0$, where $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the vacuum permittivity. The rigid half-plane is pressed into the coating to the depth $\Delta_0 = 0.01h$ = 0.05 mm and continues to slide over its surface with the constant velocity V, wearing the coating out. The wear process is considered to be finished at $t = t_w$, when the contact stress become negligible compared to its peak value ($100\% \cdot p(t_w) / \max_{t\in(0,\infty)} p(t) = 1\%$). The time t_w is called the time of coating wear by an amount Δ_0.

The effect of the sliding velocity V on the main characteristics of the contact is illustrated by Figs. 2–5 presenting the plots of $u_w(t)$, $T(h,t)$, $p(t)$, $I_0(t)$. The curves marked by ***1*** are calculated for V = 50 mm/s ($\hat{V} = 0.2$), marked by ***2*** are for V = 100 mm/s ($\hat{V} = 0.4$), ***3*** for V = 150 mm/s ($\hat{V} = 0.6$). It can be seen from Fig. 2 that the bigger sliding velocities leads to the larger maximal temperature $T_{\max} = \max_{t\in[0,t_w]} T(h,t)$ at the contact interface. The larger temperatures are responsible for the greater coating wear rate (Fig. 3). At the same time, the peak contact stresses $p_{\max} = \max_{t\in[0,t_w]} p(t)$ are almost independent on the sliding velocity V (Fig. 4). The electric current $I_0(t)$ through the piezoelectric interlayer (Fig. 5) achieves significant magnitude during the narrow time interval after the initial $t = 0$. But then $I_0(t)$ changes sign, and time dependence of I_0 obtain the same behavior as the contact temperature $T(h,t)$. Moreover, the electric current $I_0(t)$ achieves peak magnitude at the same time moments as the contact temperature $T(h,t)$. This allows one to use the electric signal from the piezoelectric interlayer electrodes to monitor the temperature on the sliding contact.

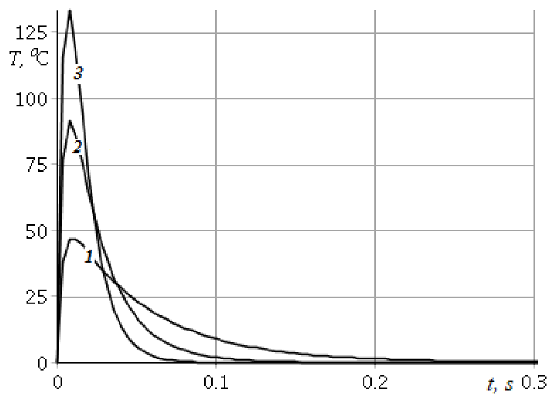

Fig. 2. Contact temperature $T(h,t)$

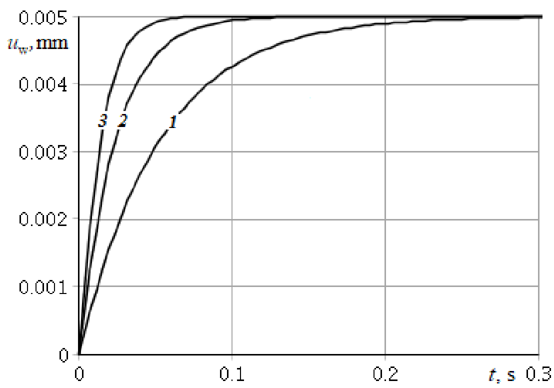

Fig. 3. Coating wear $u_w(t)$

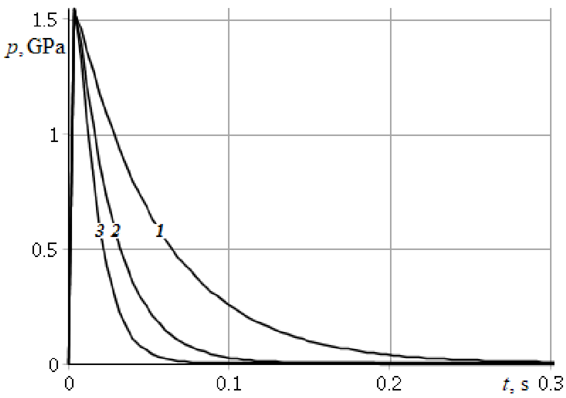

Fig. 4. Contact pressure $p(t) = -\sigma_{zz}(h,t)$

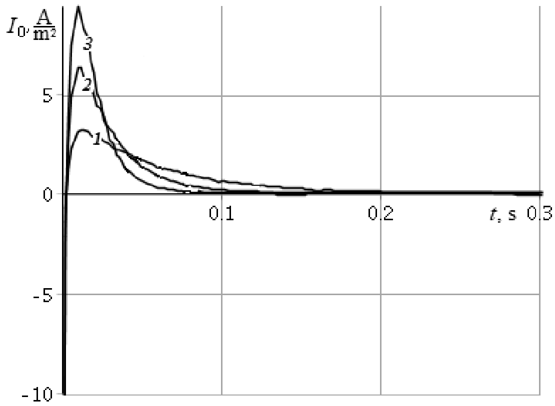

Fig. 5. Electric current $I_0(t)$ through the unit cross-section of the piezoelectric interlayer

The time dependence of the main contact characteristics is quantitatively illustrated by the Table 1. The provided data shows that multiplying the sliding velocity V by three one will get T_{max} multiplied by 2.83, I_0 by 2.90 раза, while the coating wear time t_w will be the original value divided by 4.17.

Table 1. Dependence of the main contact characteristics on the sliding velocity V

Sliding velocity V, mm/s	Maximal contact temperature T_{max}, s	Coating wear time t_w, s	Peak electric current through the unit cross-section of the piezoelectric interlayer I_0, A/m^2
50	47.2	0.25	3.26
100	91.0	0.11	6.36
150	133.7	0.06	9.44

The results above obtained were obtained for a predefined $\Delta(t)$ which means kinematically imposed loading. In wear contact problems (see, in particular, [19]), it makes sense to consider both kinematically and statically imposed loading. In this view, the next section also describes the case when a predefined pressure $\sigma(t)$ acts on the coating.

6. Contact parameters control

During the tribological devices operation one may need to control the main contact characteristics, such as the contact stress, temperature or coating wear, to prevent failures. Above, the formulas were obtained for calculation of the main characteristics of sliding contact: the temperature $T(h,t)$, stress $\sigma(h,t)$ and wear $u_w(t)$, expressed in terms of the indentation $\Delta(t)$ of the coating by the rigid half-plane I and the potential difference $V_0(t)$ on the piezoelectric interlayer B electrodes. In applications, for example, during the grinding and polishing the tool penetration $\Delta(t)$ should satisfy some requirements to prevent damage of a workpiece. In this view, on need to select the indentation law $\Delta(t)$ which will ensure, for instance: a) the workpiece load varying in the predefined range or constant, b) the temperature of the workpiece surface varying in the predefined range or constant. To fulfill such additional requirements like a) or b) occurring in tribological or machining applications, let us consider these particular applied problems, which can be formulated as inverse to the main problem considered in Section 2.

The applied problem a) can be formulated as follows. In the conditions of the main problem (Section 2) the indentation law $\Delta(t)$ of the elastic coating by the half-plane I is need to be determined in a way so the contact pressure $p(t)=-\sigma_{zz}(h,t)$ will be a predetermined function of time or constant. Using the formula (12) at $x=h$ the solution of this problem is reduced to the Volterra integral equation [20]

$$\int_0^t \Delta(t) f_\sigma^0(h,t-\tau)d\tau - \theta\int_0^t V_0(t) g_\sigma^0(h,t-\tau)d\tau = \frac{(1-2\nu)h}{2\mu(1-\nu)}\sigma(t) \qquad t>0, \tag{38}$$

where $\sigma(t)$ is the predefined function of time, $f_\sigma^0(h,t)$, $g_\sigma^0(h,t)$ are defined by (19), (20).

The integral equation (38) solution with respect to $\Delta(t)$ then can be obtained by the Laplace integral transform in the form

$$\Delta(t) = -\frac{(1-2\nu)h}{2\mu(1-\nu)}\int_0^{\tilde t} \sigma(\tau t_\kappa)\varphi_\sigma(\tilde t-\tau)d\tau + \theta V_0(t), \qquad \tilde t = t/t_\kappa \tag{39}$$

$$\varphi_\sigma(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{R(\zeta)}{t_\kappa N_\sigma(h,\zeta)} e^{\zeta t} d\zeta. \tag{40}$$

Calculation of the contour integral in (27) for $\varphi_\sigma(t)$ gives the formula

$$\varphi_\sigma(t) = (1+\eta)\delta(t) - \hat V(1-k_w)H(t) + \hat V\Phi(t) \tag{41}$$

$$\Phi(t) = -\int_0^t \frac{\exp(-1/4\tau)}{\sqrt{\pi\tau^3}}d\tau, \qquad \Phi(t)=O(t^\alpha) \quad \text{at } t\to 0,\ \alpha>0,$$

where $\delta(t)$ and $H(t)$ are, respectively, the Dirac delta function and Heaviside function.

By substituting (28) to (26), the expression for $\Delta(t)$ in terms of $\sigma(t)$ is obtained

$$\Delta(t) = -\frac{(1-2\nu)h}{2\mu(1-\nu)}\left[(1+\eta)\sigma(t) - (1-k_w)\hat V\int_0^{\tilde t}\sigma(\tau t_\kappa)d\tau + \hat V\int_0^{\tilde t}\sigma(\tau t_\kappa)\Phi(\tilde t-\tau)d\tau\right] + \theta V_0(t). \tag{42}$$

Thus, to make the contact stress develop according to some function $\sigma(t)$, one need to maintain the indentation law $\Delta(t)$ according to formula (42). If the contact stress $\sigma(t)$ needs to be constant during the wear process, i.e. $\sigma(t)=\sigma_0 H(t)$, $\sigma_0 = const$, then the formula (42) takes the form

$$\Delta(t) = -\frac{(1-2\nu)h}{2\mu(1-\nu)}\sigma_0\left[(1+\eta)H(t) - (1-k_w)\hat V\tilde t + \hat V\int_0^{\tilde t}\Phi(\tilde t-\tau)d\tau\right] + \theta V_0(t). \tag{43}$$

It should be noted that application of the potential difference $V_0(t)$ to the piezoelectric interlayer will result in expansion or shrinkage of the interlayer, so the indentation law $\Delta(t)$ should be changed accordingly to maintain the same $\sigma(t)$.

The inverse problem b) is formulated as follows: in the conditions of the main problem (Section 2) the law $\Delta(t)$ of the elastic coating indentation by the half-plane I is to be determined in a way so the contact temperature $T(t)$ will be the predefined function of time $T(t)=T_0 H(t)$, $T_0 = const$.

In this case the indentation law $\Delta(t)$ is determined from the Volterra integral equation which is obtained from (10) at $x=h$

$$\int_0^t \Delta(t) f_T^0(h,t-\tau)d\tau - \theta\int_0^t V_0(t) g_T^0(h,t-\tau)d\tau = \frac{1+\nu}{1-\nu}\alpha\frac{h}{\hat V}T(t) \qquad t>0, \tag{44}$$

where $f_T^0(h,t)$, $g_T^0(h,t)$ are defined by (19), (20).

The integral equation (44) solution with respect to $\Delta(t)$ is obtained by the Laplace integral transform in the following form:

$$\Delta(t)=\frac{1+\nu}{1-\nu}\frac{\alpha h}{\hat{V}}\int_0^{\tilde{t}}T(\tau t_\kappa)\frac{\varphi_T^*(\tilde{t}-\tau)}{2\sqrt{\pi(\tilde{t}-\tau)^3}}d\tau+\theta V_0(t),\qquad t>0 \tag{45}$$

$$\varphi_T^*(t)=-(1+\eta)F_2(t)-2(1-k_{\mathrm{w}})\hat{V}tF_1(t)+2\hat{V}\sqrt{\pi t^3}F_3(t) \tag{46}$$

$$F_1(t)=1+2\sum_{n=1}^{\infty}\exp\left(-n^2\frac{1}{t}\right)$$

$$F_2(t)=1+4\sum_{n=1}^{\infty}\frac{t-n^2}{t}\exp\left(-n^2\frac{1}{t}\right) \tag{47}$$

$$F_3(t)=\frac{2}{\sqrt{\pi t}}\sum_{n=0}^{\infty}(-1)^n\exp\left(-\left(n+\frac{1}{2}\right)^2\frac{1}{\tau}\right).$$

Here the functions $F_k(t)$ $k=1-3$ have the following asymptotic properties

$$F_{1,2}(t)=1+\mathrm{O}(t^{\alpha})\ \text{ at } t\to 0,\quad \alpha>0 \tag{48}$$

$$F_3(t)=\mathrm{O}(t^{\beta})\ \text{ at } t\to 0,\quad \beta>0,$$

where α,β are arbitrary positive numbers.

In case of $T(t)=T_0H(t)$, $T_0=const$, i.e. to maintain the constant contact temperature $T(t)$, the indentation law $\Delta(t)$ is given by the formula

$$\Delta(t)=\frac{1+\nu}{1-\nu}\frac{\alpha h}{\hat{V}}T_0\left[(1+\eta)\frac{F_2(\tilde{t})}{\sqrt{\pi\tilde{t}}}-\frac{2}{\pi}\hat{V}(1-k_{\mathrm{w}})\sqrt{\pi\tilde{t}}\cdot F_4(\tilde{t})+\hat{V}F_5(\tilde{t})\right]+\theta V_0(t),\ t>0 \tag{49}$$

$$F_4(t)=1+\frac{1}{\sqrt{t}}\sum_{n=1}^{\infty}\int_0^t\frac{\exp\left(-n^2\frac{1}{\tau}\right)}{\sqrt{\pi\tau}}d\tau \tag{50}$$

$$F_5(t)=2\sum_{n=0}^{\infty}(-1)^n\int_0^t\frac{\exp\left(-\left(n+\frac{1}{2}\right)^2\frac{1}{\tau}\right)}{\sqrt{\pi\tau}}d\tau.$$

The functions $F_{4,5}(t)$ express the following asymptotic behavior

$$F_4(t)=1+\mathrm{O}(t^{\alpha})\ \text{ at } t\to 0,\quad \alpha>0 \tag{51}$$

$$F_5(t)=\mathrm{O}(t^{\beta})\ \text{ at } t\to 0,\quad \beta>0,$$

where α,β are arbitrary positive numbers, while the function $F_2(t)$ is given by (47) and satisfies (48).

Thus, the indentation law $\Delta(t)$ can be selected by the required temperature $T(t)$ using the formula (45). For the temperature to be constant $T(t)=T_0H(t)$, the half-plane I should have indentation $\Delta(t)$ proportional to $t^{-1/2}$ at small $t>0$. This fact represent a theoretical interests though cannot be directly implemented to the real system in pure form.

After all, it should be noted that the optimal indentation law $\Delta(t)$ may need to account more than one parameter of the contact. This require more general problems to be formulated and solved rather than one of these particular problems provided here.

7. Conclusion

The obtained formulas show the possibility of controlling the sliding contact parameters. This can be achieved by adjustment either the indentation of the coating by the rigid body or the potential difference on the piezoelectric interlayer electrodes.

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