On the satellite's electrodynamic attitude stabilization

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Abstract

The paper deals with a satellite in a circular near-Earth orbit. The problem under consideration is the satellite's attitude stabilization. The control algorithms for the spacecraft electromagnetic parameters, which allow to stabilize the spacecraft attitude position in the orbital frame are obtained. The direct Lyapunov method and the developed approach for the Lyapunov functions construction are used, the sufficient conditions of the asymptotic stability of the satellite programmed motion in the orbital coordinate system are obtained. The analytical treatment of the problem is based on nonlinear differential equations of the satellite's attitude motion.The disturbing effect of the gravitational moment is taken into account.

At the same time the software complex was designed for comprehensive analysis of problems concerning the satellite attitude dynamics in geophysical fields. The report describes the complex as a complete software product.

1 Electro-dynamical attitude control system

The paper deals with a satellite in a circular near-Earth orbit. The problem under consideration is the satellite's attitude stabilization. The satellite is equipped with electrically charged shield with the total charge Q and possesses the intrinsic magnetic moment $\underline{I}$. The charge center position, determined by vector $\underline{\rho}_0$ in the frame, rigidly connected with the satellite, and the intrinsic magnetic moment $\underline{I}$ are under control. When the spacecraft is moving through the Earth's magnetic field with the magnetic induction $\underline{B}$ the interaction of the shield charge with the geomagnetic field results in Lorentz forces excitation. The principal moment of these forces with respect to the satellite's mass center may be approximated by the formula [1]

$$\underline{M}_L = \underline{P} \times \underline{T}, \tag{1}$$

where $\underline{P} = Q\underline{\rho}_0$, $\underline{T} = \underline{v}_C \times \underline{B}$, $\underline{v}_C$ is the satellite's relative velocity with respect to the Earth's magnetic field. The value of vector $\underline{B}$ in these formulas coincide with the value of vector $\underline{B}$ in the satellite's mass center.

The magnetic interaction between the satellite and the Earth's magnetic field causes the magnetic torque

$$\underline{M}_M = \underline{I} \times \underline{B}, \tag{2}$$

The mentioned torques of electro-dynamical interaction between the satellite and the Earth's magnetic field influences significantly on the satellite's attitude dynamics and can be used for the construction of the satellite attitude control systems. In these conditions it is appropriate to use parametric control for the satellite's attitude stabilization. So the satellite's electromagnetic parameters, namely the intrinsic magnetic moment $\underline{I}$ and electrostatic charge moment of the first order $\underline{P}$ are considered as the controlled quasiperiodic functions. In the stated problem the Lorentz and magnetic torques are variable in accordance with the laws of control prescribed to vectors $\underline{P}$ and $\underline{I}$.

It is well known that the magnetic moment (2) may be used for the attitude stabilization of a satellite. Such magnetic control systems may be successfully used on the satellites working for a long time because they are rather simple, possesses high reliability and don't need for the consumption of some working material. But these systems have some specific features which restrict their opportunities. As it follows from (2), the magnetic moment is orthogonal to vector $\underline{B}$. So it is impossible to construct the control magnetic moment directed along vector $\underline{B}$.

The electrostatic attitude control system was proposed in [2] and developed in [3] and [4]. This system exploits the stabilizing effect of the Lorentz torque. The usage of this moment don't need the consumption of any working material, don't need to move any heavy bodies and exhibits such advantages as the simplicity of the control law, reliability, economical operation, little mass. Moreover, the main components of described control system may be used not only for attitude stabilization of a satellite, but also for its electrostatic radiation screening. The convenience of this method is proved analytically and verified by computer modeling. But, the specific feature of the systems, exploiting $\underline{M}_L$ for the satellite's stabilization is that vector $\underline{B}$ is orthogonal to vector $\underline{T}$. So it is impossible to construct the control Lorentz moment $\underline{M}_L$ directed along vector $\underline{T}$.

Comparing the mentioned above specific features of magnetic and electrostatic systems and the following from them disadvantages of these systems, one can easily notice that these disadvantages may be avoided by constructing the united electro-dynamical attitude control system, simultaneously exploiting the moments $\underline{M}_L$ and $\underline{M}_M$. Indeed, the disadvantages rising from the existence of such directions along which is possible uncontrolled rotation of the satellite, vanishes as soon as we unite $\underline{M}_L$ and $\underline{M}_M$ in one electro-dynamical attitude control system. This fact can be explained by taking into consideration that vector $\underline{T}$ is orthogonal to vector $\underline{B}$ and consequently there are no such directions along which uncontrolled rotation of the satellite is possible.

2 Analytical and numerical results based on nonlinear mathematical model

It is proved in this paper that under some conditions there exist control algorithms for the satellite electrodynamic parameters that ensure the satellite attitude stabilization. These algorithms were obtained in explicit forms of functions depending on attitude variables with quasi-periodic coefficients. So the control algorithms for the satellite electromagnetic parameters, which allows to stabilize the satellite attitude position in the orbital frame are obtained.

With the use of numerical analysis of characteristic numbers it was proved that there exists a domain of parameters of the satellite and its orbit which ensures the total stability of the direct equilibrium position [5].

The direct Lyapunov method and the developed approach for the Lyapunov functions construction are used, the sufficient conditions of the asymptotic stability of the satellite direct equilibrium position in the orbital coordinate system are obtained [6]. The disturbing effect of the gravitational moment is taken into account. The obtained conditions allows to ensure the rational choice of the parametric control coefficients depending on the parameters of the satellite and its orbit. Moreover the paper contains the development of parametric control method by using the variable multiplication factors.

The problem of one-axis electrodynamic satellite stabilization in the indirect position in the orbital frame is solved. On the base of the Lyapunov functions method, the conditions of the asymptotic stability of the satellite equilibrium position are obtained [7].

The method of electrodynamic attitude control was also successfully used for the problems of the stabilization of a satellite rotation in the orbital coordinate system. The programmed motion under which the axis of the satellite's dynamic symmetry is along the local vertical, and the satellite rotates around this axis is stabilized. On the base of the Lyapunov functions method, the sufficient conditions for asymptotic stability of the stabilized programmed motion in the presence of gravitational disturbing torque are obtained [8]. These conditions permit to provide a rational choice of the coefficients of parametric control depending on the parameters of the satellite and its orbit.

At the same time the accurate preliminary analysis of the forces acting upon a satellite and the construction of thoroughly analyzed mathematical model of the problem is the urgent task for achievement the required stabilization accuracy.

3 Software complex

Therefore the software complex was designed for comprehensive analysis of problems concerning the satellite attitude dynamics in geophysical fields. Through the use of computer algebra methods the problem of constructing the simplest possible, but correct mathematical model that ensures the prescribed accuracy is solved. As a result the octupole approximation of the Earth's magnetic field was accepted and the stability of the satellite's stabilized orientation was proved analytically and confirmed by computer modeling.

The report describes the complex as a complete software product, designed primarily to assess the effectiveness of the method of electrodynamic stabilization of satellites, combined with the possibility of using some of its modules to address a wide variety of local problems.

One of these local problems is the problem of choice rather simple and correct mathematical model of the Earth's magnetic field which ensures the prescribed accuracy. Unfortunately, we can not confine ourselves to simple models that are usually used in the preliminary analytical analysis of the problem, and the refinement of models does not allow keeping the task within the capabilities of man. For example, to take into account the impact of the Earth's magnetic field on the dynamics of the satellite, we need to know the induction vector $\underline{B}$ as a function of the radius-vector of the point of near-Earth space. However, in reality, the Earth's magnetic field have a very complicated structure, so that this functional dependence is absent in the analytical form. Hence the need for mathematical modeling of the Earth's magnetic field in those tasks in which accuracy requirements does not allow us to restrict ourselves to the simplest well-known approximations (such as dipole magnetic field). The developed set of programs aimed at solving these problems by using computer algebra capabilities implemented in Maple, and further use of the constructed models for the numerical analysis.

For the first time the algorithm for analytical multipole representation of the Earth's magnetic field with an arbitrary degree of accuracy was constructed. The mathematical justification of this method was given in [9] and developed in [10], [11]. The algorithm and software for Maple, implements symbolic computation, allowing building the analytical expressions of multipole tensors of arbitrary rank, expressing them in terms of Gauss coefficients, is developed. A program that allows obtaining vector $\underline{B} = \sum_{n=1}^{N} \underline{B}^{(n)}$ in any finite approximation is constructed.

The near-Earth space was divided into regions in which the account of a finite number of vector $\underline{B}$ multipole components is correct due to the chosen criteria of accuracy. This allows us to establish the necessary and sufficient number of terms in the multipole expansion of vector $\underline{B}$, which provides the required accuracy of finding this vector at any point in the satellite orbit with the given radius and inclination.

A program for evaluation the system of differential equations in Rodrigues-Hamilton parameters describing the satellite's attitude motion is constructed. A program for numerical solution of the Cauchy problem in the satellite electrodynamic stabilization and further numerical analysis of this solution and its visualization is written. As a result of integration of differential equations of the satellite's attitude motion one can see the time history graphics of Rodrigues-Hamilton parameters, the angles of orientation of the satellite (shown in Fig.1), and the magnitudes of control and disturbing moments.

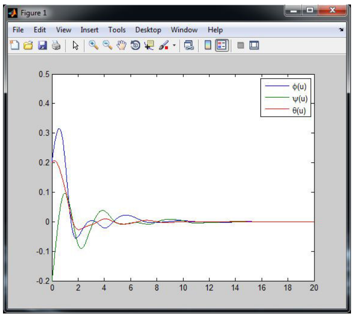

Figure 1

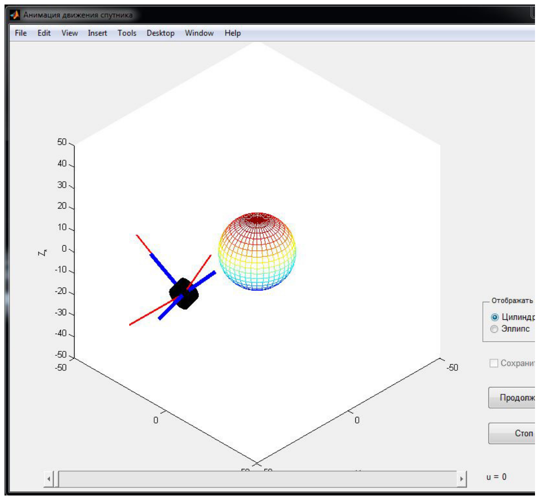

Figure 2

The designed software complex allows watching the 3D-animation of the process of satellite's electrodynamic attitude stabilization. The fragment of this animation is shown in Fig.2. Thin axes correspond to the orbital frame and thick axes correspond to the satellite's principal central axes of inertia.

In addition, the complex contains a number of software modules to automate the following procedures: the linearization of differential equations, the finding the stability regions for the solutions of differential equations in the parameters planes, the optimization of the electrodynamic control [4].

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