Fatigue corrosion criterion of complex mechanical systems

Robert A. Arutyunyan
Robert.Arutyunyan@paloma.spbu.ru

Abstract

The reliability problem of complex mechanical systems composed of some number of elements with the crack type defects which growth rate is essentially governed by the processes of corrosion fatigue is considered. Taking into account that the reliability of a system as a whole depends on the reliability of individual elements and the way of there connection, the systems with the elements connected parallel and in series are considered. For each system the reliability function based on the Poisson's and Weibull's distribution is defined. The fatigue corrosion criterions are formulated and the corresponding fatigue curves are constructed. It is shown that the reliability of the system with the parallel connected elements is superior to the reliability of a system with elements connected in series.

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Under complex system the modern technical designs which failure is possible as a result of growth of corrosion fatigue cracks are considered. We assume that the system in an initial condition contains n defective elements. Deficiency of an element is characterized by existence of cracks. Thus, in an initial condition the system contains the cracks which average size allows to consider their behavior from positions of fracture mechanics. The kinetic equation of a crack growth rate in the inert environment is described by Peris-Erdogan's equation [1]. For crack growth in the corrosion environment the kinetic equation based on the description of chemical processes of dissolution of metal in a tip of a crack is formulated. Thus it is considered that the stress intensity factor is responsible for acceleration of corrosion processes.

When formulating the kinetic equation for the fatigue corrosive crack growth, the following consideration was taken into account [2, 3]. The involvement of an aggressive environment in fatigue crack growth depends on a complex interaction between chemical, mechanical and metallurgical factors which leads to the intensification of the crack growth rate. To describe these experimental effects the total crack extension rate under corrosion fatigue conditions is approximated by a simple superposition of the crack growth rate in an inert atmosphere and the crack growth rate due to aggressive environment

$$\frac{dl}{dN} = C(\Delta K)^m + \frac{d\gamma}{dN}, \tag{1}$$

where ΔK is stress intensity factor range, C, m are material variables, l is current value of the crack length, γ is the current depth of crack due to corrosive degradation of material at the tip of a crack, N are the loading cycles.

To formulate the crack extension rate due to aggressive environment it was assumed that the corrosive degradation of material is described by the first order chemical equation, where the stress intensity factor is considered to control the chemical reaction

$$\frac{d\gamma}{dN} = F(\Delta K)N^{\beta}, \tag{2}$$

where F is a function of ΔK and β is a constant. Further we will consider the power relation for the function F: $F(\Delta K) = K_1(\Delta K)^{\alpha}$ (K_1 and α are a constants).

Taking $\alpha = m$ and introducing (2) into (1), we will receive the following kinetic equation

$$\frac{dl}{dN} = (\Delta K)^m (C + K_1 N^{\beta}). \tag{3}$$

Further, we will consider the propagation of a small through thickness crack in a large plane specimen, for which $\Delta K = \Delta\sigma\sqrt{\pi l}$ ($\Delta\sigma$ is stress range). Introducing this value of stress intensity factor range into (3) we will have

$$\frac{dl}{dN} = (\Delta\sigma)^m \pi^{m/2} l^{m/2} (C + K_1 N^{\beta}). \tag{4}$$

For $\Delta\sigma = const$, $l = l_{0i}$ at $N = 0$ the relation of the crack growth length l_i in i-th element follows from equation (4) as

$$l_i(N) = \left[\frac{2-m}{2}(\Delta\sigma)^m \pi^{m/2}\left(CN + K_1\frac{N^{\beta+1}}{\beta+1}\right) + l_{0i}^{\frac{2-m}{2}}\right]^{\frac{2}{2-m}}. \tag{5}$$

The relation (5) is used to formulate the probabilistic fatigue corrosion criterion of complex mechanical systems with damaged elements [4, 5]. We consider that the size distribution of initial cracks is random. The number of cycles to fracture is also random. Under the influence of the cyclic stresses, a crack start and grow. The fracture of a system is a result of achievement a crack length of some critical value. At traditional approach to calculation on durability of complex technical systems the damaged state of system as a whole isn't considered. Such method of calculation can lead to overestimate of survival of system as the probability of survival of a system from many elements with defects is much lower than probability of survival of a separate element [5, 6]. The way of connection of elements is essential also: in series, in parallel and with reservation.

Let elements of system interact so that failure of any of them leads to system failure. In this case the elements of the system are connected in series (Fig. 1).

Figure 1

No-failure operation of system is the random event equal to crossing of independent events - no-failure operation of each of elements. Probability of no-failure operation of system we will receive according to the multiplication theorem for independent events. Reliability function in this case looks like as

$$R(N) = \exp\left[-n(N)G(N)\right]. \tag{6}$$

Let $l_{0i} \le l_i \le l_*$ ($i = \overline{1,n}$) is random selection of Poisson's distribution, where l_* is the critical length of a crack:

$$G(l) = \frac{e^{-\lambda l_{0i}} - e^{-\lambda l_i}}{e^{-\lambda l_{0i}} - e^{-\lambda l_*}}. \tag{7}$$

Let's N_i is the number of cycles before achievement i-th cracks of critical length, then $G(N)$ is distribution function of N_i and according to (5), (7) can be written as

$$G(N) = \frac{e^{-\lambda l_{0i}} - e^{-\lambda l_i(N)}}{e^{-\lambda l_{0i}} - e^{-\lambda l_*}}. \tag{8}$$

Introducing (8) into (6), we will receive

$$R(N) = \exp\left\{-n(N)\frac{e^{-\lambda l_{0i}} - e^{-\lambda l_i(N)}}{e^{-\lambda l_{0i}} - e^{-\lambda l_*}}\right\}. \tag{9}$$

For the given reliability level $R = R_*$, from relation (9) and (5) follows the fatigue corrosion criterion for system connected in series

$$(\Delta\sigma)^m\left(CN + K_1\frac{N^{\beta+1}}{\beta+1}\right) = \frac{2}{(2-m)\pi^{m/2}}\left[\left(\frac{1}{\lambda}\ln\left(\frac{1}{B}\right)\right)^{\frac{2-m}{2}} - l_{0i}^{\frac{2-m}{2}}\right], \tag{10}$$

where $B = e^{-\lambda l_{0i}} + \dfrac{e^{-\lambda l_{0i}} - e^{-\lambda l_*}}{n}\ln R_*$.

The case of the general reservation with k elements connected in parallel and n elements connected in series is shown on Fig. 2.

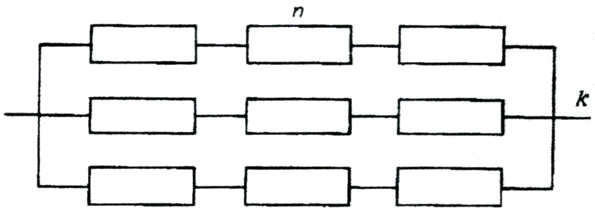

Figure 2

For this system reliability function and fatigue criterion (for reliability level $R = R_*$) is found as

$$R(N) = 1 - \Big[1 - \exp\big[-n(N)G(N)\big]\Big]^k, \tag{11}$$

$$(\Delta\sigma)^m\left(CN + K_1\frac{N^{\beta+1}}{\beta+1}\right) = \frac{2}{(2-m)\pi^{m/2}}\left(\left(\frac{1}{\lambda}\ln\left(\frac{1}{B}\right)\right)^{\frac{2-m}{2}} - l_{0i}^{\frac{2-m}{2}}\right), \tag{12}$$

where $B = e^{-\lambda l_{0i}} + \dfrac{e^{-\lambda l_{oi}} - e^{-\lambda l_*}}{n}\ln\left(1 - (1-R_*)^{\frac{1}{k}}\right)$.

Calculations were carried out also for the reliability function given by Weibull's distribution. In this case we have:

for the system connected in series

$$R(N) = \exp\left[-n(N)\frac{e^{-\lambda l_{0i}^{\varphi}} - e^{-\lambda l_i^{\varphi}(N)}}{e^{-\lambda l_{0i}^{\varphi}} - e^{-\lambda l_*^{\varphi}}}\right], \tag{13}$$

and for the system with general reservation

$$R(N) = 1 - \left[1 - \exp\left[-n(N)\frac{e^{-\lambda l_{0i}^{\varphi}} - e^{-\lambda l_{i}^{\varphi}(N)}}{e^{-\lambda l_{0i}^{\varphi}} - e^{-\lambda l_{*}^{\varphi}}}\right]\right]^{k}. \quad (14)$$

In relations (13) and (14) φ is constant.
The corresponding fatigue corrosion criteria have the form

$$(\Delta\sigma)^m \left(CN + K_1\frac{N^{\beta+1}}{\beta+1}\right) = \frac{2}{(2-m)\pi^{m/2}}\left(\left(\frac{1}{\lambda}\ln\left(\frac{1}{B}\right)\right)^{\frac{2-m}{2\varphi}} - l_{0i}^{\frac{2-m}{2}}\right), \quad (15)$$

where $B = e^{-\lambda l_{0i}^{\varphi}} + \dfrac{e^{-\lambda l_{0i}^{\varphi}} - e^{-\lambda l_{*}^{\varphi}}}{n}\ln R_*$,
and

$$(\Delta\sigma)^m \left(CN + K_1\frac{N^{\beta+1}}{\beta+1}\right) = \frac{2}{(2-m)\pi^{m/2}}\left(\left(\frac{1}{\lambda}\ln\left(\frac{1}{B}\right)\right)^{\frac{2-m}{2\varphi}} - l_{0i}^{\frac{2-m}{2}}\right), \quad (16)$$

where $B = e^{-\lambda l_{0i}^{\varphi}} + \dfrac{e^{-\lambda l_{0i}^{\varphi}} - e^{-\lambda l_{*}^{\varphi}}}{n}\ln\left(1 - (1-R_*)^{\frac{1}{k}}\right)$.
The theoretical curves plotted according to the relations (10), (12), (15), (16) will be applied to describe the behavior of the considered complex mechanical systems with elements connected in series and in general reservation. Comparative analysis of these curves will allow to choose the criterion of corrosion fatigue strength which is the closest to the experimental results known in the world scientific literature.

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Robert A. Arutyunyan, Universitetskii pr., 28, Faculty of Mathematics and Mechanics Sankt-Petersburg State University, Sankt-Petersburg, Petrodvoretz, 198504, Russia.