Bodies with cracked coatings

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Abstract

Boundary-value problem is considered, which describes the behavior of materials with coatings under conditions of vibrational harmonic actions. The coatings are considered mutually contacting, deformable two-dimensional objects with polytypic properties and parameters averaged over thickness (in particular, this can be thin-walled plates), forming a layer on a three-dimensional deformable substrate.

In general, the coatings are assumed to be cracked. Such a model of coating arises traditionally in seismology and in aircrafts after long using.

An algorithm for constructing solutions of such boundary-value problems is described using the topological approach, which has not been studied in this general formulation so far. The problem is reduced to investigating a new class of sets of pseudodifferential equations, for which the determinant of the matrix function of the kernel symbol is identically zero. The conventional methods of investigation are inapplicable to such systems. In this study, we derive approximate formulas for solving these sets of equations.

Materials described in the boundary-value problems formulated above can possess a complicated set of the properties, depending on the physical and mechanical characteristics of the substrate and coatings, their shapes, and the mutual arrangement of the surface. They also depend on the conditions of interaction of coatings with the substrate and with each other in all areas of contact, as well as on the external actions upon the substrate-coating system. In particular, the properties can include dissimilarity in the concentration of stresses under coatings and in the localization of strain.

1 Topological method

There is represented the method of research and boundary problem solution for block structures. This approach introduces the topological rendering of block element's method, developed in works [1, 2, 3, 4, 5]. It makes the use of this method not only convenient, but it also provides the prospect of future development with involvement of profoundly developed methods of topology and theory of manifolds. Particularly it proves the possibility of wide variety of block elements' carrier forms.

Semianalytic method of block element as opposed to just computative, allowed to reveal set of earlier unknown properties of boundary problems in block structures. Thus, in works [6, 7] existence of natural viruses is revealed, in [8] the possibility of energy confinement and other process parameters that lead to their abnormal behaviour is discovered. We can continue with examples [9]. The exposition of topological method, though it is a repetition of algorithm of block elements method, brings it nearer to the new potentials in accordance with use of deep theoretical developments in topology.

1. Lets believe, that we examine linear boundary problem for the differential equation system in partial derivatives for the block structure, consisting of blocks, which occupy three-dimensional area Ω_b, $b = 1, 2, \ldots, B$, whose deformable environments have multitype physical and mechanical characteristics. The blocks contact with each other, one part of their bounds can be loose. The blocks can be limited and unlimited, they can occupy as simply connected, so multiply connected areas with piecewise-smooth boundary $\partial\Omega_b$.

Lets introduce several topologies. The first one connects with Ω_b areas, occupying the blocks irrelatively to the boundary problem. We suppose, that block structure consists of contact blocks and presents all-in-one, it may also be multiply connected. Examining block areas in metric space, lets bring into each of them topology, induced with open environs $\upsilon_{\nu b}$, $\nu = 1, 2, \ldots, \nu_b$ [10, 11]. The block environs can get crushed or consolidated. The largest open environ in the block is $\upsilon_b = \cup\upsilon_{\nu b}$. It is a consolidation of all open environs. It represents the interior of the block, and its locking $\bar{\upsilon}_b$ gives the block with bounds. Topological spaces, constructed in each block, are subspaces T_{1b} of topological space of the whole block structure T_1. Lets accomplish compactification of space T_1, by adding the environs of infinitely remote point, if the block structure consist it.

Lets enter into consideration functions from the space $\mathbf{H}_s$in every open space $\upsilon_{\nu b}$. Linear normed space induces topology, for example, with open functions balls. Lets examine set of functions of $\mathbf{H}_s$ in every open environ $\upsilon_{\nu b}$, as in a carrier, and construct topological structure, taking as set the open balls

$$\|\varphi\|_{\mathbf{H}_s} < \varepsilon$$

Thus the open set of functions $\Upsilon_{\nu b}$ is formed on every open set $\upsilon_{\nu b}$.

The totality of open environs $\Upsilon_{\nu b}$ forms in the areas $\Omega_{\nu b}$ topological structure of subspace T_{2b}, which is a part of topological structure T_2, including open sets of all blocks. According to environs T_2 construction the spaces T_1 and T_2 are isomorphic.

Since topological space T_1 is regular in construction, it with its every subspace allows partition of unity. Lets perform the unity partition of compacted topological space T_1, and therefore of every subspace T_{1b}, with non-intersecting connected open covering, which we will mark as usual for the sake of brevity as $\upsilon_{\lambda b}$, $\lambda = 1, 2, \ldots, \lambda_b$. It shows that in virtue of isomorphism the partition of space unity T_1 involves equivalent partition of space unity T_2. Let's construct topological manifold M_{1b} in topological space T_{1b}, for this we will enter local systems of coordinates, maps and atlas in every covering $\upsilon_{\nu b}$ Their consolidation results multifold M_1. Let's name open coverings $\upsilon_{\lambda b}$ as interiors of manifolds M_{1b} and their closings $\bar{\upsilon}_{\lambda b}$ as orientable manifolds with edge M_{1b}, after entering of local system of coordinate and tangent bundle of bounds. We got a totality of oriented infinitely smooth multifold M_{1b} with edge. In virtue of isomorphism the functions, forming T_2 form also multifold M_2 and M_{2b}. We can examine them as objects of topological space T_2, so as the functions on multifold $M_1 = \cup M_{1b}$. Let's indicate through Θ the additional areas Ω till the whole space R^3 as $\Theta = R^3 \backslash \Omega$, which does not contain carriers of block structure. Let's perform the covering areas Θ with open areas $\theta_{\mu r}$, which may contact with some multifolds at the place, where their bounds is loose and which we will name as null multifolds. $\upsilon_{\lambda b}$. As a result the covering of the whole space R^3 will be performed with open non-intersecting multifolds.

2. Lets enter into consideration a boundary problem for the system of differential equation in partial derivative at the area, occupying with block structure.

$$\mathbf{K}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A^b_{spmnk}\varphi_{bp,\overset{(m)}{x_1}\overset{(n)}{x_2}\overset{(k)}{x_3}} = g_b(\mathbf{x}) \tag{1}$$

$$s = 1, 2, \ldots, P_b, \quad A^b_{sqmnk} = const, \quad \varphi_b = \{\varphi_{b1}, \varphi_{b2}, \ldots, \varphi_{bP}\}, \quad b = 1, 2, \ldots, B$$

$$\varphi = \{\varphi_s\}, \quad \varphi(\mathbf{x}) = \varphi(x_1, x_2, x_3), \quad \mathbf{x} = \{x_1, x_2, x_3\}, \quad \mathbf{x} \in \Omega_b$$

The following boundary conditions are set on the common contacting bounds

$$\mathbf{R}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b + \mathbf{R}_d(\partial x_1, \partial x_2, \partial x_3)\varphi_d = \sum_{m=1}^{M_1}\sum_{n=1}^{N_1}\sum_{k=1}^{K_1}\sum_{p=1}^{P}\left[B^b_{spmnk}\varphi_{bp,\overset{(m)}{x_1}\overset{(n)}{x_2}\overset{(k)}{x_3}} + B^d_{spmnk}\varphi_{dp,\overset{(m)}{x_1}\overset{(n)}{x_2}\overset{(k)}{x_3}}\right] = f_{bds} \quad (2)$$

$$s = 1, 2, \ldots, s_{b0} < P, \quad \mathbf{x} \in \partial\Omega_b \cap \partial\Omega_d, \quad M_1 < M, \quad N_1, < N, \quad K_1 < K$$

$$b,\, d = 1, 2, \ldots, B$$

In the case, if the area Ω_d is null area, only the term with index, b the loose bounds, remains in the formula under the sign of sum. The boundary problem studies in spaces of temperate generalized function $\mathbf{H_s}(\Omega)$, described [1, 2]. Let's introduce in consideration a Cartesian product of topological space $T_1 \times T_2$. Let's undergo its mapping concerning the rule: T_1 is mapped identically to itself; mapping of T_2 is introduced by the form $\mathbf{K}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b$, which transmits a vector φ_b, from M_{2b} to an assigned vector g_b to M_{2b} concerning (2).

For solution of a boundary problem it is necessary to find an image of this mapping.

2.1. Start with a setting of a case, when the ratios of differential form (1) are constant in every block. Proceeding in a space R^3 to functions from T_2, belonging to $\mathbf{H_s}(\Omega)$, to topological dual of Fourier – images, receive the relations, which are called the functional equation type (2),

$$\mathbf{K}_b(\alpha)\mathbf{\Phi}_b = \iint\limits_{\partial\Omega_b} \boldsymbol{\omega}_b - \mathbf{G}_b(\alpha), \quad \mathbf{G}_b(\alpha) = \iiint\limits_{\Omega} \mathbf{g}_b(\mathbf{x}) \exp i \langle \alpha \mathbf{x} \rangle \, dx_1 x_2 x_3$$

$$\mathbf{K}_b(\alpha) \equiv -\mathbf{K}_b(-i\alpha_1, -i\alpha_2, -i\alpha_3) = \left\| k_{bnm}(\alpha) \right\|$$

$$\alpha\mathbf{x} = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3, \quad b = 1, 2, \ldots, B$$

where the symbols of this work are saved. In process of fulfilled buildings the Stokes' integral was used on multifold with edge, which led to appearing of exterior form $\boldsymbol{\omega}_b$. From developed functional equations we find vectors representation φ_b, involving unknown values on blocks' boundaries. For finding the solutions of boundary values φ_b at conjugated or free boundaries of blocks' region $\partial\Omega_b$ it is necessary to unify the boundaries of contacts or adjacent coverings $\bar{\upsilon}_b$ and $\bar{\upsilon}_d$, or adjacent coverings $\bar{\upsilon}_d$ and $\bar{\theta}_d$ if a block has a free boundary. For this purpose a factor is built – a space topology $T_1 \times T_2$, which fulfills a factor of equivalent block boundaries or the same, equivalent boundaries of topological closed-sets, under condition (2). Let's mark, that in this case in space T_1 an uprising of coverings by uniting them by the boundary, and in T_2 space a pseudodifferential equation is formed, which carries out the union of the covering elements of this topological space. This condition is far more complicated than the traditionally stated examples of factor-spaces in literature of topology [11], but only after its realization a factor-topology in our case is formed. A suitable algorithm of conducting this process is described in [2]. Let's suppose that 2 blocks $\bar{\upsilon}_b$ and $\bar{\upsilon}_d$ are in contact. Let's examine only a part of boundary of their interaction irrespective to the contacts with other blocks. Then the procedure, which describes the building of resolving interactions for solving primary boundary problem, includes the following actions:

- Analyzing the multifold with edge M_{2b} and M_{2d} in local coordinate axes [10];

- A choice of the most effective curvilinear coordinate system for fulfillment of automorphism [12, 13];
- Fulfillment of a differential factorization of matrix-function $\mathrm{K}_b(\alpha)$ and $\mathbf{K}_d(\alpha)$ [14];
- Computing the Leray residue form [2];
- Building of pseudodifferential equations [2];
- Extracting from pseudodifferential equations by the demanded condition (2) of integral equations [2, 13, 14];
- The solution of integral equations;
- Adding the found solutions into the integral representation of a solution of boundary problem.

$$\varphi_b = \mathbf{F}^{-1}\mathbf{K}_b^{-1}(\alpha)\left[\iint\limits_{\partial\Omega_b} \boldsymbol{\omega}_b - \mathbf{G}_b(\alpha)\right]$$

- Operator $\mathbf{F^{-1}}$ of Fourier transforms.

In this case after the enlargement of manifold M_{1d} and M_{1b} by uniting into $M_{1b} \cup M_{1d}$, building of isomorphic manifolds $M_{2b} \cup M_{2d}$ will follow.

2.2. The case of variable coefficients in differential form (1) is different from the case, which is discussed above, that the covering is dictated by the features of its coefficients. It should be as small in size, that analyzing at it differential form (1) could be related to a category, which has constant coefficients. Than every such covering becomes the block element, but which has the coefficients. In this case at the boundary of such blocks the boundary conditions interface solutions should be formulated, similarly (2), dictated by the demand of software smoothness of solutions, it is possible, theirs differential quotients or gradients and other forms at the boundary. After that, all actions for such block structure for the given boundary problem, which are stated in the previous paragraph.

2.3. In case of nonlinear boundary problem a research and its solution can be fulfilled by using the Newton-Kantorovich's method [15], which demands for its realization at each step a converse of some linear not uniform boundary problems with variable coefficients, which, it is shown in previous paragraphs, is feasible for the method.

Therefore, an offered method can be researched a wide range of boundary problems from different fields. It should be noticed, that usage of this method allows building of analytical representation of boundary problem solution, and it is of extreme importance, for example, for analyzing a wave processes and revealing of different irregular conditions in a multiparameter processes.

Note 1. The simplified schemes of multifold buildings are stated at 2.1 in terms that the blocks have one map. Without any effort using this scheme a research can be fulfilled also in the cases, when there are several maps. In this case it is necessary to "increase" the quantity of blocks, putting for each one map. In this case it is necessary to form additional conditions of the type (2), providing the continuation of solutions from one block to another concerning the demanding dictating by the boundary problem conditions of persistence.

Note 2. Application to topological approach in the method of block element allowed setting important property of possibility of wide chose of its carriers which can be absolutely arbitrary open sets. For all these cases exists an algorithm providing the process of getting the answer bounding sum.

Note 3. It is perfectly clear that given block structure can contain different types of heterogeneity as cracks, inclusions and vesicle. In his case block elements can be built with the method of relative (virtual) division in block structure without crossing border of heterogeneity. In the case when three-dimensional block structure contains deformed blocks of smaller sizes, for example, plates or shells coupling of such elements has its own peculiarity and the algorithm stated above cannot be used.

2 Application for bodies with cracked coatings

Here are expounding theoretical essentials of seismic prediction, which lean upon the concept of grade of concentration tensions from lithospheric plate's mechanical effect, which are susceptible to external influences of different nature and which are slow drifting. There are different approaches to its realization in a number of domestic and foreign works.

However, their feature is considerable simplification of concerned problems, the employment of extremely simple mathematical apparatus. Such approach in complex can't encompass just as a huge variety of processes in the deep layers of the earth and on its surface, and the properties of deformable medium, which make up the lithosphere plates, and the types of their faults.

Let's remark, that information about deep properties is extremely miserly and in a number of cases unknown. In this connection we must find such approach to the simulation of lithospheric plate's behaviour, which would allow, as needed, to interact with the model in conversational mode, including new discovered information.

Some of this information could be considered, such as the presence of inhomogeneities of lithospheric plates, refinement types fault's types, whether they are through or of limited depth, the presence of internal cavities and cracks in the tectonic plates. Equally important is the information about the properties of the lithospheric plate's contact with the upper mantle, the asthenosphere.

An important issue is the possible impacts of the voltage source on all types of lithospheric plates, including those from the daytime surface-wind and sedimentary, radiation, electromagnetic.

Attempts to account these factors, which were made in a number of approaches, usually do not allow to do it in the complex, resulting in their dissociation, and the study of individual tasks for each of them. It leads not only to the loss of exactness, but also to the loss of important properties of decisions' behavior, boundary associated with the manifestation of the natural virus.

The latest were found only in connection with investigations of the effects of several factors on the solution of boundary problems considered together.

In connection with above-listed, the most suitable for studying of the tectonic plates behavior, is a method of block element, which has a topological basis, successfully developed and applied in the SSC RAS and Kuban University.

Below there is an example of the simplest models of lithospheric plates, which demonstrate the implementation of the mechanical concept. The basis for the mechanical concept of seismicity evaluation is the identification of stress concentration zones in the lithospheric plates as another precursor of seismicity, upon which we can judge about the possible consequences of seismic events, their locations, and in some situations, about the probable time of their occurrence.

3. Let us separately consider on the topological approach of the block structures theory with the availability of different size block elements. In contrast to analyzed in work [13, 16] block structures with blocks of the same dimension, this has its own specificity. It can be studied with different approaches. As a rule, such cases happen when the three-dimensional deformable bodies are in contact with the two-dimensional, such as plates or shells. These contacts are covering three-dimensional bodies' shells, the presence of technology inclusions from the plates in three-dimensional bodies, etc. The covering may be laminated. The part of the coating can be split, or have cracks. Just such a situation occurs in the tectonic plates that contain faults, both interior and emergent to the surface. Cracked coverage take place in aviation, where levels of allowable defects of aircraft are defined, allowing the continuation of their safe exploitation. In science of material developed theories, which explain the strength of high quality polished metal surfaces with the presence of surface tension, which is simulating by thin coating. In theory of nancovering appear problems of studying of strength of such facilities, including with the availability of such cracks.

3.1. During topological study of block structure, consisting of the above two-and three blocks, two approaches are possible. The first approach includes a primary topological study of separately each block structure – two-dimensional and three-dimensional, by the method described in [2, 3, 13, 16], considering all irregularities, cracks and breaks. Then goes the operation, which is called the construction of the quotient topology, which consists in the identification of two-dimensional boundary of three-dimensional element block with a median of two-dimensional surface of the coating. By this means are building pseudodifferential equations and integral equation for the construction of the boundary values of the considered boundary value problem.

The second approach consists of preliminary construction quotient topology bi-block elements – three-dimensional and two-dimensional, with a subsequent study of a new topological object containing different size components and matching different size limits.

The second approach requires proper consideration of all the topological features of such facilities, especially during the construction of the tangent bundle of the border, the introduction of the local coordinate systems, map and atlas of the manifold.

4. For example, consider the boundary value problem for a plate, as a simple model that gives mixed-block structure in contact with the three-dimensional substrate. We take a plate consisting of different types of horizontal contact pieces, which can also be cracked, located on a deformed half-space. Particularly, this is the simplest model of lithospheric plates, modeled by Kirchhoff plates, consisting of parts of horizontally-oriented units, plates, and containing the faults of arbitrary geometry. At present time experimental data on the movements of tectonic plates successfully determined by the high precision receivers GPS/GLONASS. Based on this model, we will consider a plate with a two-dimensional variety of faults with the edge. The area occupied by the plate, let mark Ω. Divide the plate into units based on the requirements, keeping in each block uniformity and consistency properties. Carry out, in addition, the division into blocks along faults or cracks, even if the crack crosses the same type of unit. Let B – the number of resulting blocks after such breakdown. Then we have $\partial\Omega = \cup\partial\Omega_b$, $b = 1, 2, 3, \ldots, B$. Block boundaries $\partial\Omega_b$ will be of the different types. A part $\partial\Omega_{b1}$ of each boundary $\partial\Omega_b$ can provide rigid contact with the adjacent block, the other part $\partial\Omega_{b2}$ can be free from the stress and bending, and the third- $\partial\Omega_{b3}$ may be the crack, i.e., in general $\partial\Omega_b = \cup\partial\Omega_{br}$, $r = 1, 2, 3$.

Let's save the traditional shift designations $\mathbf{u} = \{u_1, u_2, u_3\}$ and mechanical parameters designations [17, 18]. Below u_1, u_2 – the shift of the plate points with horizontal directions of the middle surface, u_3 perpendicularly to it. In this case the formula of the differential component of the operator

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right)\mathbf{u}_b - \varepsilon_{5b}\mathbf{g}_b = \varepsilon_{5b}\mathbf{t}_b$$

has the form [8, 9]

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right)\mathbf{u}_b = \begin{Vmatrix} \psi_{11}u_{1b} & \psi_1 u_{2b} & 0 \\ \psi_{12}u_{1b} & \psi_{22}u_{2b} & 0 \\ 0 & 0 & \psi_{33}u_{3b} \end{Vmatrix}$$

$$\psi_{11} = \frac{\partial^2}{\partial x_1^2} + \varepsilon_{1b}\frac{\partial^2}{\partial x_2^2} + \varepsilon_{4b}, \quad \psi_{12} = \varepsilon_{2b}\frac{\partial^2}{\partial x_1 \partial x_2}, \quad \psi_{22} = \frac{\partial^2}{\partial x_2^2} + \varepsilon_{1b}\frac{\partial^2}{\partial x_1^2} + \varepsilon_{4b},$$

$$\psi_{33} = \varepsilon_{3b}\left(\frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2} + \frac{\partial^4}{\partial x_2^4}\right) + \varepsilon_{4b}$$

$$\mathbf{R}_b(-i\alpha_1, -i\alpha_2)\mathbf{U}_b = -\begin{Vmatrix} \xi_{11}U_{1b} & \xi_{12}U_{2b} & 0 \\ \xi_{12}U_{1b} & \xi_{22}U_{2b} & 0 \\ 0 & 0 & -\xi_{33}U_{3b} \end{Vmatrix} \tag{3}$$

$$\xi_{11} = \alpha_1^2 + \varepsilon_{1b}\alpha_2^2 - \varepsilon_{4b}, \quad \xi_{12} = \varepsilon_{2b}\alpha_1\alpha_2, \quad \xi_{22} = \alpha_2^2 + \varepsilon_{1b}\alpha_1^2 - \varepsilon_{4b}$$

$$\xi_{33} = \varepsilon_{3b}(\alpha_1^2 + \alpha_2^2)^2 + \varepsilon_{4b}$$

$$\mathbf{U} = \mathbf{F}_2\mathbf{u}, \quad \mathbf{G} = \mathbf{F}_2\mathbf{g}, \quad b = 1, 2, \ldots, B$$

Here

$$\varepsilon_{1b} = 0.5(1 - \nu_b), \quad \varepsilon_{2b} = 0.5(1 + \nu_b), \quad \varepsilon_{3b} = \frac{h_b^2}{12}$$

$$\varepsilon_{4b} = \omega^2\rho_b\frac{1 - \nu_b^2}{E_b}, \quad \varepsilon_{5b} = \frac{1 - \nu_b^2}{E_b h_b}$$

$$g_{1b} = \mu_b\left(\frac{du_{1b}}{dx_3} + \frac{du_{3b}}{dx_1}\right), \quad g_{2b} = \mu_b\left(\frac{du_{2b}}{dx_3} + \frac{du_{3b}}{dx_2}\right) \tag{4}$$

$$g_{3b} = \lambda_b\left(\frac{du_{1b}}{dx_1} + \frac{du_{2b}}{dx_2} + \frac{du_{3b}}{dx_3}\right) + 2\mu_b\frac{du_{3b}}{dx_3}, \quad x_3 = 0$$

Here the designations for the plates are: λ, μ – the parameters of Lame, ν – the coefficient of Poisson, E – the module of Young, h – thickness, ρ – density, ω – vibration frequency. $\mathbf{g}_b = \{g_{1b}, g_{2b}, g_{3b}\}$, $\mathbf{t}_b = \{t_{1b}, t_{2b}, t_{3b}\}$ – the vectors of the contact tension and external stress, operating along the axis x_3 in the area Ω_b $\mathbf{F}_2 \equiv \mathbf{F}_2(\alpha_1, \alpha_2)$ – two-dimensional operator of Fourier's transformations.

Various boundary conditions depend on the type of the boundary part of each block. Thus, under accepted designations, the boundary conditions for the case of joint unlocking in the contact zone, i.e. free rotation on the boundary around the axis x_1 have the form:

$$M = -D\left(\frac{\partial^2 u_3}{\partial x_1^2} + \nu\frac{\partial^2 u_3}{\partial x_2^2}\right) = 0, \quad D = \frac{Eh^2}{12(1 - \nu^2)} \tag{5}$$

For the case when the plate edges are allowed to shift freely along the axis x_3 the boundary condition has the formula:

$$Q = -D\left(\frac{\partial^3 u_3}{\partial x_2^3} + (2 - \nu)\frac{\partial^3 u_3}{\partial x_1^2 \partial x_2}\right) = 0 \tag{6}$$

In case of firm fixity it is necessary to demand that the shifts in the direction to the axes are equivalent to zero

$$u_1 = 0, \quad u_2 = 0, \quad u_3 = 0; \tag{7}$$

To block the middle surface turn around the axis x_1 it is necessary to demand to carry out of the conditions

$$\frac{\partial u_3}{\partial x_2} = 0. \tag{8}$$

The formulae for the normal and circumferential tension components on the boundary are given according to the ratios

$$N_{x_2} = \frac{E}{1-\nu^2}\left(\frac{\partial u_2}{\partial x_2} + \nu\frac{\partial u_1}{\partial x_1}\right), \quad T_{x_1x_2} = \frac{E}{2(1+\nu)}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)$$

As the deformed baseboard – substratum on which the plate-covering are situated that is described by the boundary value problem (3), various models can be accepted. This can be deformed semispace, layer, multilayer semispace, including anisotropic, elastic-plastic media. In all enumerated cases the ratios between the tension on the stratified medium surface g_{kb}, $k = 1,2,3$ and the shifts u_k, $k = 1,2,3$ have the form (2) with the properties

$$\mathbf{u}(x_1,x_2,x_3) = \frac{1}{4\pi^2}\iint\limits_{-\infty}^{\infty} \mathbf{K}(\alpha_1,\alpha_2,x_3)\mathbf{G}(\alpha_1,\alpha_2)e^{-i\langle\alpha,x\rangle}d\alpha_1 d\alpha_2 \tag{9}$$

$$\langle\alpha,x\rangle = \alpha_1 x_1 + \alpha_2 x_2$$

$$\mathbf{K} = \|K_{mn}\|, \quad m,n = 1,2,3, \quad \mathbf{K}(\alpha_1,\alpha_2,0) = O(A^{-1}), \quad A = \sqrt{\alpha_1^2+\alpha_2^2} \to \infty$$

$K_{ks}(\alpha_1,\alpha_2,x_3)$ – analytical functions of two complex variables α_k meromorphic in particular, their numerous examples are given in [19, 20].

These ratios are called dominant functions.

When the equations describing the behavior of the baseboard medium are known, the elements of the matrix-function $\mathbf{K}(\alpha_1,\alpha_2,0)$ can be calculated. When such equations are lacking, the dominant functions can be received by experiment.

5. For example, consider the scalar case of vertical vibrations of the plate. In this case the functional equation (3) of the boundary problem for this case, which is given for the plate described above as for the varifold with the boundary, splits for each block and is given as the ratio [1]

$$\mathbf{R}_b(-i\alpha_{1b},-i\alpha_{2b})U_{3b} \equiv (\varepsilon_{3b}(\alpha_{1b}^2+\alpha_{2b}^2)^2+\varepsilon_{4b})U_{3b} = \int\limits_{\partial\Omega_b} \omega_b + \mathbf{F}_2(g_{3b}+t_{3b}) \tag{10}$$

$$b = 1,2,\ldots,B$$

Here ω_b is the exterior form, the part of the presentation, which has the form

$$\omega_b = \varepsilon_{3b}e^{i\langle\alpha,x\rangle}(\beta_1 dx_1 - \beta_2 dx_2)$$

$$\beta_1 = \frac{\partial^3 u_{3b}}{\partial x_2^3} - i\alpha_2\frac{\partial^2 u_{3b}}{\partial x_2^2} - \alpha_2^2\frac{\partial u_{3b}}{\partial x_2} + i\alpha_2^3 u_{3b} + 2\frac{\partial^3 u_{3b}}{\partial x_1^2\partial x_2} - i\alpha\frac{\partial^2 u_{3b}}{\partial x_1^2}$$

$$\beta_2 = \frac{\partial^3 u_{3b}}{\partial x_1^3} - i\alpha_1 \frac{\partial^2 u_{3b}}{\partial x_1^2} - \alpha_1^2 \frac{\partial u_{3b}}{\partial x_1} + i\alpha_1^3 u_{3b}$$

The block boundary, as said above, can have various contact properties with the adjacent blocks or be free. This property must be included in the presentation of the pseudodifferential equation. To build it, the functional equation coefficient (10) roots should be found, after that the automorphism requirement is carried out with regard to the functional equation, the calculations of the residue form Leray [2, 3].

As a result the pseudodifferential equations come out.

Take that one of the block boundary parts, Ω_{br} is a straight line.

In this case the group of the pseudodifferential equations which is built on this part has the form

$$\mathbf{F}_1^{-1}(\xi_1^r) \left\{ \int\limits_{\partial\Omega_{br}} \eta_1 e^{i\alpha_1^r x_1^r} dx_1^r + \int\limits_{\partial\Omega_b \backslash \partial\Omega_{br}} \omega_b + \mathbf{F}_2(g_{3b} + b_{3b}) \right\} = 0,$$

$$\mathbf{F}_1^{-1}(\xi_1^r) \left\{ \int\limits_{\partial\Omega_{br}} \eta_2 e^{i\alpha_1^r x_1^r} dx_1^r + \int\limits_{\partial\Omega_b \backslash \partial\Omega_{br}} \omega_b + \mathbf{F}_2(g_{3b} + b_{3b}) \right\} = 0, \quad (11)$$

$$\eta_j = D^{-1} M_r - D^{-1} Q_r - (\alpha_{2j-}^3 + \nu\alpha_1^2) \frac{\partial u_{3r}}{\partial x_2^r} + i\alpha_{2j-} \left[\alpha_{2j-}^2 + (2-\nu) \right] u_{3r},$$

$$j = 1, 2, \quad \alpha_2 = \alpha_{2j-}, \quad \xi_1^r \in \partial\Omega_{br}$$

Here $\mathbf{F}_1^{-1}$ is the inverse operator for Fourier's one-dimensional transformation.

In the integrands it should be accepted

$$\alpha_{21-} = -i\sqrt{(\alpha_1^r)^2 - \sqrt{\frac{\varepsilon_4}{\varepsilon_3}}}, \quad \alpha_{22-} = -i\sqrt{(\alpha_1^r)^2 + \sqrt{\frac{\varepsilon_4}{\varepsilon_3}}}$$

accordingly. If the boundary is not a straight line, the object of consideration is the minor zone of this boundary. Other groups of the pseudodifferential equations on the other boundary zone look similarly. The characteristic property of the principal operator is that it contains all the types of the boundary conditions which the plate on the boundary can have when there are vertical vibrations, analytical expressions of which are given as the ratios (5)–(8). When we copy out all pseudodifferential equations for every unit of board and for every block, inserting in them corresponding boundary constraints and solving extracted from pseudodifferential equations integral equation, we'll get performance of solution in every plain block of correlation.

$$u_{3b} = \mathbf{F}_2^{-1} \left[\mathbf{R}_b(-i\alpha_{1b}, -i\alpha_{2b}) \right]^{-1} \left\langle \int\limits_{\partial\Omega_b} \omega_b + \mathbf{F}_2(g_{3b} + b_{3b}) \right\rangle \quad (12)$$

Obtained performance u_{3b} (10) of two-dimensional block structure is is equated u_{3b} (9) with value $x_3 = 0$ of third-dimensional block structure and finally integral equation is obtained for definition of surface stress between overlapping and substructure that carry information about accumulation of stresses in lithosphere plate from vertical effect. Represented nonvector variant without special effort is transferred to the vector one, described by two equations in (3). Together with decision of complete boundary problem for set of equations (3) there is a possibility on the ground of monitoring information, which includes GPS/GLONASS receiving set, to estimate accumulation of stresses in models of lithosphere plates . More precise model of area is obtained with the use of deformed third-dimensional lithosphere plates that are horizontally disposed on deformed foundation with appliance of approaches presented in works [21].

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