Hierarchical sequence of models and deformation peculiarities of porous media saturated with fluids

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Abstract

The joint model of two-phase flow and porous space deformation is analyzed in this paper. The model includes the conservation laws written for mixture components and constitutive equations in the form of generalized Hook's law. The system factorization according to the small parameter allows to separate out the hierarchycal sequence of models corresponding to the specific medium deformation conditions. The zero-order approximation of the model presents the medium with invariable volume and pore pressure equation uncoupled from one for rock matrix deformations. An analytic solution of the system in cylindrical coordinates with non-zero shear stresses that may produce rock failure is obtained.

1 Introduction

The theory of fluid saturated porous media is one of important and highly developing scientific branches. This is a consequence of its wide practical significance (water cleaning, ground water movements, coal deposits or oil and gas fields exploitation etc.) and theoretical value caused by the absence of the universal model of these processes. The complexity of such theory creation is defined by the requirement of taking into account joint rock matrix deformations and flows of different fluids (liquid or gas) within it.

One of the first successful attempts to formulate a coupled deformation model was made by M.Biot in 1941 [1]. Almost at the same time the theory of physical processes in saturated soil was developed by J.I. Frenkel [2]. In Biot's model the one-phase saturation is considered and deformations are determined for elementary representative volume. The relationship between the full stresses in volume, strain and fluid pressure has a form of Hook's law with additional term associated with pore pressure expansion. The pore pressure value is defined from the non-stationary one-phase filtration equation which includes the volumetric deformation with correspondent poroelastic constants.

The following Biot's papers (for example, [3]) were devoted to the poroelastic theory adaptation and application to the acoustical tasks and contain description of two compressible and one shear waves in porous materials. In Russia the theory of acoustical wave propagation in saturated porous media was developed in [4]-[6].

An obvious step in poroelastic theory generalization is introduction of multiphase saturation. The problems relating to the joint porous skeleton deformation and two-phase fluids flow are of important practical value, in particular in gas-and-oil industry. W.Brutsaert [7] presented the poroelastic theory generalization for porous media with liquid and gas flow in pores. The model took into account that both fluids might have different pressure. The constitutive equations which state the relationship between full stresses, pore pressures and strains are written in the form of generalized Hook's law and conditions of joint deformations. The equations of medium motion are derived from the variational principle by the differentiation of system's lagrangian with respect to the velocity vector components. The poroelastic model was used for acoustic waves attenuation problem in porous medium. It was shown that in accordance with this theory three types of compressional wave and one type of shear wave existed in porous material.

The porous media saturated with two phases was considered in the series of articles [8, 9], but with equal pressures in phases. The equations of motion were derived from Hamilton's variational principle but the explicit form of the stress, pore pressure and strain relations were not presented. The stress tensor was presented by strain invariants in a general form. The method of coefficients determination in linearized poroelastic two-phase model by Biot's coefficients and volume fractions (saturation) was proposed. The model was applied to the analysis of acoustic waves with different frequencies.

The model of saturated porous medium, where equations were stated, probably, in the most general form was published in [10]. The full system of equations with assumption of united pressure in liquid phases and some other essential simplifications is transformed to J. Berryman's model [8]. After additional assumption of one-phase flow it is shown that the system can be reduced to Biot's model. It should be noted that only in this case the stress tensor in solid phase is explicitly presented in the article through the medium deformations and poroelastic constants. The resulting system of linear equations is used by the authors for solving problems in acoustics.

For stress-strain analysis of rocks around mines or boreholes, the acoustic approximation is not enough and in this case the non-linearity of equations and nonhomogeneous of medium should be taken into account. In the present work the model of two-phase flow in deformable porous media which consists of conservation laws and constitutive equations in the form of generalized Hook's law is considered.

Since the rock matrix deformation and fluid flows are characterized by different scales, the small parameters can be introduced into the transport equations. As a result models hierarchy describing porous medium dynamics is obtained. With the help of small parameter factorization, the applicability of existing models can be determined. In particular, it was shown that Biot models were obtained by taking into account non-sequence members.

As a first step we analyze the equations of zero-order model approximation which corresponds to the incompressible medium. In this situation the filtration equations are uncoupled from deformation equation. We show that in 3D case for nonhomogeneous pore pressure distribution on cylindrical surface, the analytical solution for displacements which produce the shear stresses exists. The amplitude of stresses can be sufficient for medium destruction.

2 Mathematical model of joint motion

The equations of dynamics of the system under consideration (fluids and solid skeleton) are based on the mechanics of interpenetrating continua [4]. Since further we examine the isothermal system, it is sufficient to consider the mass and momentum conservation equations. The mass conservation law in this case has a form (phase transition is absent)

$$\frac{\partial \rho_i}{\partial t} + div(\rho_i \underline{v}_i) = 0, \;\; i = o, w, s; \tag{1}$$

here $\rho_i = \alpha_i \rho_i^c$ - partial density, α_i - volume phase concentration, ρ_i^c - density of the pure (compact) phase, indexes o, w, s designate oil, water and rock matrix correspondingly,

$\underline{v}_i = \frac{\partial \xi_i}{\partial t}$ - phase velocity, $\underline{\xi}_i$ - phase points displacements. Momentum balance equations for liquid phases and for the whole system can be written as

$$\frac{\partial \rho_i \underline{v}_i}{\partial t} + \sum_j \frac{\partial (v_{ij} \rho_i \underline{v}_i)}{\partial x_j} = div\left(\alpha_i \boldsymbol{\sigma^i}\right) - \sum_{k \neq i} r_{ik} (\underline{v}_i - \underline{v}_k) - \frac{1}{3} tr \boldsymbol{\sigma^i} \nabla \alpha_i + \rho_i \underline{g},\ i = o, w; \quad (2)$$

$$\frac{\partial}{\partial t}\left(\sum_{i=o,w,s} \rho_i \underline{v}_i\right) + \sum_j \frac{\partial}{\partial x_j}\left(\sum_{i=o,w,s} v_{ij} \rho_i \underline{v}_i\right) = div(\boldsymbol{G}) + \rho_m \underline{g}, \quad (3)$$

where $k = o, w, s$, index m denotes mixture, $j = 1, 2, 3$ - coordinate axis indexes, $\boldsymbol{\sigma}^i$ - stress tensor of the i-th phase, $\rho_m = \sum_i \rho_i$, $\boldsymbol{G} = \sum_i \alpha_i \boldsymbol{\sigma}^i$ - general stress tensor of the whole mixture, r_{ik} - the coefficient of interphase friction. As a rule, the friction between liquid phases is not taken into account therefore we will not describe the appropriate coefficients. Following the generalized Darcy law [11] the friction coefficients between liquids and rock matrix can be represented as $r_{is} = \alpha_i^2 \mu_i / (k f_i)$, where μ_i is the phase viscosity, $f_i(\alpha_i)$ is the relative phase permeability, k - is permeability of porous medium. The third member in the right part of (2) is the Jukovskiy-Rahmatulin force modeling the reaction of rock matrix and second liquid phase as geometric constraint. Thermodynamics equations of state for pure phases are presented in the form

$$\rho_i^c = \rho_i^c(p_i, T),\ i = o, w,\ \rho_s^c = \rho_s^c(T),$$

here T is the temperature, $p_j = -\frac{1}{3} tr \boldsymbol{\sigma}^j$ - phase pressure. Since the averaged deformation ε^s does not describe the rock matrix deformation on the microscale level it seems for us incorrect to introduce the relation $\rho_s^c = \rho_s^c(p_s)$.

We use constitutive equations for liquid phases in the form of Stock's law (in principle, the more general constitutive relation can be used):

$$\boldsymbol{\sigma}^i = (-p^i + \lambda_i tr \dot{\boldsymbol{\varepsilon}}^i) \boldsymbol{E} + 2\mu_i \dot{\boldsymbol{\varepsilon}}^i,$$

$$\dot{\boldsymbol{\varepsilon}}^i = \frac{1}{2}\left(\nabla \otimes \underline{v}_i + (\nabla \otimes \underline{v}_i)^T\right) \boldsymbol{E}.$$

Then neglecting the surface effects we consider $p_w = p_o = p$. We assume that porous matrix is the isotropic linear elasticity body. In this case, following the works of M.Biot, J. Berryman and Wei-Cheng Lo, for the model with one pressure, the mechanical constitutive equations in the form of generalized Hook's law can be deduced:

$$\boldsymbol{G} = 2\mu_m \boldsymbol{\varepsilon}^s + (\lambda_m e - C_m \zeta) \boldsymbol{E}, \quad (4)$$

here $\boldsymbol{E}$ is unit tensor, $\zeta = \zeta_w + \zeta_o$, $\zeta_i = div[\alpha_i(\underline{\xi}_s - \underline{\xi}_i)],\ i = o, w$ is the parameter of phase compressibility relative to rock matrix (the increment of fluid content). The joint deformation condition in this case turns to

$$p = M_m \zeta - C_m e. \quad (5)$$

The second joint deformation condition is evident:

$$\alpha_w + \alpha_o + \alpha_s = 1. \quad (6)$$

The definition of the parameters $\mu_m,\ \lambda_m,\ C_m,\ M_m$ versus component properties and composition of the system is a very complex task that is outside the scope of this article. Thereby we obtain complete joined model of the elastic rock matrix deformation and two-phase filtration with one pressure (1)-(6). It is worth to note that inspite of equal pressure in liquid phases the velocities in general are different.

3 The equations in dimensionless form

The final form of equations is quite complex and for the further analysis it is useful to consider it in dimensionless variables. Let's introduce first of all the character system scales: d will be the pore scale and L is the problem scale. Then let's denote $P_0^s = \rho_{rock} gH$ as a character geomechanical pressure (it is weight of the rock or overburden load) and $P_0^{w,o} = P_0^f = \rho_f gH$ is the character fluid pressure (the mud fluid pressure, in particular). Here H is the collector depth. For liquid-liquid-solid system $P_0^s \sim P_0^{w,o}$; in system liquid-gas-solid $P_0^g \neq P_0^f \sim P_0^s$. The character filtration velocity is equal to $V_0 = \frac{d^2}{\mu_f L} P_0^f$ and the character displacements values are $\tilde{\underline{\xi}}_s = d \cdot \underline{\xi}_s$, $\tilde{\underline{\xi}}_i = L \cdot \underline{\xi}_i$, $i = w, o$. Finally, the dimensionless viscosity coefficients are defined by the relations: $\tilde{\mu}_w = \mu_f \cdot \mu_o$ where μ_f is the mud fluid viscosity. As a result, the dimensionless equations of motion have the form

$$\frac{\partial \rho_i}{\partial t} + div(\rho_i \underline{v}_i) = 0, \ i = o, w; \tag{7}$$

$$c_\rho \frac{\partial \rho_s}{\partial t} + c_\rho \delta \cdot div(\rho_s \underline{v}_s) = 0, \tag{8}$$

$$\begin{aligned} &Np \cdot \delta^4 \frac{\partial \rho_i \underline{v}_i}{\partial t} + Np \cdot \delta^4 \sum_j \frac{\partial (v_{ij} \rho_i \underline{v}_i)}{\partial x_j} = \\ &= div \left\{ \alpha_i \left[\left(-p_i + \delta^2 \lambda_i tr \dot{\boldsymbol{\varepsilon}}^i \right) \boldsymbol{E} + \delta^2 \mu_i \dot{\boldsymbol{\varepsilon}}^i \right] \right\} - \sum_{k \neq i} r_{ik} (\underline{v}_i - \underline{v}_k) - \\ &- \frac{1}{3} tr \boldsymbol{\sigma^i} \nabla \alpha_i + Np \cdot \delta^4 \rho_i \underline{g}, \qquad i = o, w, \ k = o, w, s, \ j = 1, 2, 3; \end{aligned} \tag{9}$$

$$\begin{aligned} &Np \cdot \delta^4 \frac{\partial}{\partial t} (\rho_w \underline{v}_w + \rho_o \underline{v}_o + c_\rho \delta \rho_s \underline{v}_s) + Np \cdot \delta^4 \sum_j \frac{\partial}{\partial x_j} (v_{wj} \rho_w \underline{v}_w + \\ &+ v_{oj} \rho_o \underline{v}_o + c_\rho \delta^2 v_{sj} \rho_s \underline{v}_s) = div(\boldsymbol{G}) + Np \cdot \delta^4 \cdot (\rho_w + \rho_o + c_\rho \rho_s) \underline{g}. \end{aligned} \tag{10}$$

Finally the generalized Hooks law and joint deformation conditions take the form:

$$\zeta = div \left[\alpha_w \left(\delta \underline{\xi}_s - \underline{\xi}_w \right) \right] + div \left[\alpha_o \left(\delta \underline{\xi}_s - \underline{\xi}_o \right) \right], \tag{11}$$

$$\begin{cases} \boldsymbol{G} = \{ c_\rho \lambda_m \varepsilon_1^s \boldsymbol{E} + c_\rho 2 \mu_m \boldsymbol{\varepsilon}^s + C_m \zeta \boldsymbol{E} \} \equiv \boldsymbol{\sigma}^f - Bp\boldsymbol{E} \\ p = \{ C_m \varepsilon_1^s + M_m \zeta \} \end{cases} \tag{12}$$

here $B = C_m / M_m$ is the analog of Biot-Willis parameter, $\boldsymbol{\sigma}^f = (c_P \lambda_m - C_m^2) \epsilon_1^s \boldsymbol{E} + c_P \cdot 2 \mu_m \boldsymbol{\epsilon}^s$ is the fictituous stress tensor. Beside of this the following dimensionless complexes are introduced:

$$\delta = \frac{d}{L}, \qquad c_P = \frac{P_0^s}{P_0^f}, \qquad c_\rho = \frac{\rho_0^s}{\rho_0^f}, \qquad Np = \frac{\rho_0^f P_0^f L^2}{\mu_f^2}.$$

4 The approximate models

Since the parameter δ in general case is much less than one we can consider a sequence of models. Let's analyze zero- and first-order approximation model on δ. The equations (7) is independent on δ. From equation (8) in zero-order approximations we derive that

$\rho_s = \rho_s^0$. Since we accept the hypothesis of small deformation and linear elasticity then we can represent the density alteration through the strain tensor invariants $\rho_s = \rho_s^0(1-\epsilon_1^s)$, where $\epsilon_1^s = \epsilon_{ii}^s = div\underline{\xi}_s = 0$ in this case. The momentum balance equations (9) for liquid phases lead to the classical generalized Darcy law [11]. The equation (10) for "mixture" is transferred to the classical static equation. Generalized Hook's law and joint deformation condition become simpler. Finally, the model has the form:

$$\begin{aligned} &\alpha_i \underline{v}_i = -\frac{kf_i}{\mu_i}\nabla p,\ i=o,w \qquad div(\boldsymbol{G})=0 \\ &\boldsymbol{G} = c_P\cdot 2\mu_m \boldsymbol{\varepsilon}^s + C_m\zeta\boldsymbol{E},\ p = M_m\zeta, \\ &\zeta = -div(\alpha_w\underline{\xi}_w) - div(\alpha_o\underline{\xi}_o). \end{aligned} \tag{13}$$

The mass conservation laws in the first-order model approximation doesn't undergo simplification. The momentum conservation law implies that liquids flow relative to solid matrix is described again by the generalized Darcy law:

$$\alpha_i(\underline{v}_i - \delta\underline{v}_s) = -\frac{kf_i}{\mu_i}\nabla p,\ i=o,w.$$

The momentum balance equation for the whole system is transferred to the static one. The equations (11),(12) remains the same. For the comparison with classic Buckley-Leverett filtration model [11] we write here the equations for common pressure in phases and water concentration α_w when densities $\rho_i^c,\ i=o,w$, are constant:

$$div\left\{k\left(\alpha_w\frac{f_w}{\mu_w}+\alpha_o\frac{f_o}{\mu_o}\right)\nabla p\right\} = \delta\cdot div(\underline{v}_s), \qquad \frac{\partial\alpha_w}{\partial t} + div(b_w\underline{V} + \alpha_w\delta\underline{v}_s) = 0, \tag{14}$$

here $b_i = \frac{\kappa_i}{\kappa_1+\kappa_2}$, $\kappa_i = k\frac{f_i}{\mu_i}$, $i=o,w$, $\underline{V} = -k\left(\alpha_w\frac{f_w}{\mu_w}+\alpha_o\frac{f_o}{\mu_o}\right)\nabla p$. The system of equation is the same as for Buckley-Leverett model only in the case of $\delta=0$ or $div\ \underline{v}_s = 0$. Both of these cases correspond to our zero-order approximation.

5 The analytical solution for the quasiplane deformations in cylindrical coordinates

Let's consider the case when permeability, porosity and phase concentration do not depend on spacial variables, i.e. $m_0(x) = m_0^0,\ k=k_0,\ \alpha_i = const,\ i=o,w$. In this case the equation for pore pressure will have the form: $\Delta p = 0$. Therefore, in zero-order approximation the hydrodynamics equations are uncoupled from rock matrix deformation equation and can be solved independently. Using the methodology for the presentation of the elastic equilibrium equations through the rotation vector [12] the equations of our model can be written as:

$$rot\underline{\omega} = A\nabla p,\ rot\underline{\xi}_s = \underline{\omega},\ div\underline{\xi}_s = 0,\ \Delta p = 0, \tag{15}$$

here $A = B/\mu_m$. This is the overdetermined elliptic system. The analogous systems arise in hydrodynamics and their solvability have been studied in [13].

The deformation of material in this approximation is characterized by conservation of solid matrix volume. It should be noted that, despite a strong simplification, the model includes a non-zero shear strain and stress. The analytical solutions for systems of this type are constructed in [14]-[16]. However, due to the one-dimensional approximation they could not identify this effect.

Let's consider the system (15) in cylindrical coordinates (r, φ, z). We will look for the solution of the form: $\xi_r = u(r, \varphi),\ \xi_\varphi = v(r, \varphi),\ \xi_z = w(r, \varphi)$. In this case we can derive from (15) the following system:

$$\frac{\partial}{\partial \varphi}\left[\frac{1}{r}\left(\frac{\partial (rv)}{\partial r} - \frac{\partial u}{\partial \varphi}\right)\right] = -rA\frac{\partial p}{\partial r}, \tag{16}$$

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial (rv)}{\partial r} - \frac{\partial u}{\partial \varphi}\right)\right] = \frac{A}{r}\frac{\partial p}{\partial \varphi}, \tag{17}$$

$$\left\{\frac{\partial}{\partial r}\left[r\frac{\partial w}{\partial r}\right] - \frac{\partial}{\partial \varphi}\left[\frac{1}{r}\frac{\partial w}{\partial \varphi}\right]\right\} = -rA\frac{\partial p}{\partial z}, \tag{18}$$

$$e = \frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \varphi} = 0. \tag{19}$$

It should be noted that the task for w, equation (18) is uncoupled from other displacement components. Hence the problem can be represented as the superposition of the plain task and the antiplane strain. We may term this particular case of three-dimensional stress-deformed state as the "quasiplane" deformations. In next calculations we will not consider the w component and equation for w. Let's take the harmonic function for pressure in the form:

$$p = r^n\left(C_1 cos\ n\varphi + C_2 sin\ n\varphi\right) + \frac{1}{r^n}\left(C_3 cos\ n\varphi + C_4 sin\ n\varphi\right).$$

Expressing from (19) the mixed derivatives for $rv,\ u$, we obtain from (16) the equation for u and from (17) the equation for v. After cumbersome but simple calculations, we can get solutions for the displacements as the sum of a particular solution, corresponding to the pressure $u_p,\ v_p$ and the general solution $u_0,\ v_0$ of the homogeneous equations:

$$u_p = \frac{An}{(1+n)}r^{n+1}\left(C_1 cos\ n\varphi + C_2 sin\ n\varphi\right) - A\frac{n}{(1-n)}\frac{1}{r^{n-1}}\left(C_3 cos\ n\varphi + C_4 sin\ n\varphi\right),$$

$$u_0 = \frac{C_7}{r} + \frac{1}{r^{k+1}}\left(D_1 cos\ k\varphi + D_2 sin\ k\varphi\right) + r^{m-1}\left(D_3 cos\ m\varphi + D_4 sin\ m\varphi\right),$$

$$v_p = A\frac{(n+2)}{(1+n)}r^{n+1}\left(C_2 cos\ n\varphi - C_1 sin\ n\varphi\right) - A\frac{(n-2)}{(1-n)}\frac{1}{r^{n-1}}\left(C_3 sin\ n\varphi - C_4 cos\ n\varphi\right),$$

$$v_0 = \frac{1}{r^{k+1}}\left(D_1 sin\ k\varphi - D_2 cos\ k\varphi\right) + r^{m-1}\left(-D_3 sin\ m\varphi + D_4 cos\ m\varphi\right) + rC_5 + \frac{C_6}{r},$$

here C and D with indexes are the arbitrary constants and m, n, k are the integer numbers. Direct substitution shows that this solution satisfies the equation (19). It can be calculated that shear stress in quasiplane deformation is not zero:

$$\sigma^f_{r\varphi} = \mu_m\left(\frac{1}{r}\frac{\partial u}{\partial \varphi} + r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right) =$$
$$= \frac{Bn}{2}r^n\left(-C_1 sin\ n\varphi + C_2 cos\ n\varphi\right) + \frac{Bn}{2}\frac{1}{r^n}\left(-C_3 sin\ n\varphi + C_4 cos\ n\varphi\right) +$$
$$+ \frac{2\mu_m(k+1)}{r^{k+2}}\left(-D_1 sin\ k\varphi + D_2 cos\ k\varphi\right) +$$
$$+ 2\mu_m(m-1)r^{m-2}\left(-D_3 sin\ m\varphi + D_4 cos\ m\varphi\right) - \frac{2\mu_m C_6}{r^2}.$$

The solution for shear stress has enough arbitrariness of constants for solving technical tasks.

Deformations of the material in the zero-order approximation are characterized by the conservation of the solid matrix volume and also by the presence of shear strain and stress. As follows, the approximation is interesting since even in this simple case induced in medium shear strains causing creep and rock failure can be captured.

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