Motion of a pendulum with vibrating suspension axis at unconventional values of parameters

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Abstract

In the classical papers [1,2] motion of a pendulum with vibrating suspension axis was considered in the case, when frequency of external loading is much higher than the natural frequency of the pendulum in the absence of this loading. The present paper is concerned with the analysis of pendulum's motion at unconventional values of parameters. Case, when frequency of external loading and the natural frequency of the pendulum in the absence of this loading are of the same order, is studied. Vibration intensity is assumed to be relatively low. A new modification of the method of direct separation of motions (MDSM) [3,4] is proposed to study corresponding equations, which in the considered case don't contain a small parameter explicitly. A condition of pendulum's upper position stabilization is determined for this case by its means. It is noted that in the considered range of parameters not only the effective "stiffness" of the system changes due to the external loading, but also its effective "mass". It is shown that application of the classical asymptotic methods in this case leads to erroneous results. So, the applicability range of the MDSM turns out to be broader than the one of these methods.

1 Introduction

Widely used approaches for the analysis of equations, in which a small parameter may be assigned, are the classical asymptotic methods, in particular, the method of averaging [5,6], the multiple scales method (MSM) [7] and other approaches [8,9,10]. Application of these methods for solving equations, which don't contain a small parameter explicitly, is rather cumbersome and may lead to erroneous results.

The present paper is concerned with the analysis of the applicability of the method of direct separation of motions (MDSM) [3,4] for solving such equations. A new modification of the method, which may be employed for their studying, is introduced. It is noted that, as in the case of the classical asymptotic methods, application of the modified MDSM implies an accurate account of orders of the parameters in the considered equation. A correlation between the MDSM and Ritz's method of harmonic balance [11,12], Van der Pol's method of slowly varying amplitudes [12], the classical asymptotic methods and other approaches [13] is revealed. The present study may be considered as development and continuation of the ideas, proposed in the monograph [14] for the analysis of equations, which don't contain a "natural" small parameter, by means of the Poincare method: An assumption regarding the type of the equation's solution is employed in order to introduce a small parameter.

A classical problem about the stability of a pendulum with vibrating suspension axis is considered as an illustrative example. Case, when frequency of external loading and the natural frequency of the pendulum in the absence of this loading are of the same order, is studied. Vibration intensity is assumed to be relatively low. A condition of pendulum's upper position stabilization is determined for this case. At corresponding values of the parameters it reduces to the classical condition of pendulum stabilization [1,2]. It is noted that in the considered range of parameters not only the effective "stiffness" of the system changes due to the external loading, but also its effective "mass".

It is shown that application of the MSM for solving the considered equations in the studied range of parameters leads to erroneous results. Thereby, the applicability range of the MDSM turns out to be broader than the one of the classical asymptotic methods.

2 Stabilization of a pendulum with vibrating suspension axis. The modified MDSM

Consider a classical problem about the stability of a pendulum with vibrating suspension axis in the simplest formulation, i.e. in the case of small deviations from the upper position. In this case motion of the pendulum is described by the following equation:

$$I\ddot{\varphi} - ml(g + G\Omega^2 \cos \Omega t)\varphi = 0 \tag{1}$$

Here φ is small angle of pendulum deviation from the upper position, I, m and l are the moment of inertia, the mass and the distance from the pendulum center of gravity to the axis of suspension, G and Ω are the amplitude and the frequency of vertical oscillations of the suspension axis, g is the acceleration of gravity, dot designates the time derivative.

Introducing two dimensionless parameters $\delta = \frac{mlg}{I\Omega^2}$, $\chi = \frac{G\Omega^2}{g}$ and dimensionless time $t_0 = \Omega t$, rewrite equation (1) in the form

$$\frac{d^2\varphi}{dt_0^2} - \delta(1 + \chi \cos t_0)\varphi = 0 \tag{2}$$

The case $O(\delta) = O(\chi) = 1$ is considered: the frequency of external loading and the natural frequency of the pendulum in the absence of this loading are of the same order; the amplitude of acceleration of vertical oscillations of pendulum's suspension axis is of the same order as g. In this case equation (2) doesn't contain a small parameter explicitly.

For studying equation (2) the vibrational mechanics approach [3,4] is employed. The MDSM is applied in the form, which differs from the conventional one [3,4]; the solutions are sought in the form:

$$\varphi = \alpha(t_1) + \psi(t_1, t_0) \tag{3}$$

where $t_1 = \varepsilon t_0$, $\varepsilon << 1$ is small parameter, α is "slow", and ψ is "fast", 2π - periodic in dimensionless time t_0 variable, with average zero:

$$\langle \psi(t_1, t_0) \rangle = 0$$

Here $\langle ... \rangle$ designates averaging in the period 2π on time t_0, i.e. for function $h(t_1, t_0)$ we have $\langle h(t_1, t_0) \rangle = \frac{1}{2\pi} \int\limits_0^{2\pi} h(t_1, t_0) dt_0$.

As is seen, a hypothesis regarding the type of the solution sought is used in order to introduce a small parameter ϵ. In fact, searching solution of the initial equation in the form (3), we assume that the system in the considered range of parameters performs oscillations with slowly varying characteristics. If this hypothesis is not correct, then the trivial solution or solution, which doesn't meet the sense of the problem, will be obtained. Otherwise the parameters of oscillations of the considered type will be determined.

While solving equation (2) consider variables t_1 and t_0 as independent, so that $\frac{d^2}{dt_0^2} = \frac{\partial^2}{\partial t_0^2} + 2\varepsilon\frac{\partial^2}{\partial t_1 \partial t_0} + \varepsilon^2\frac{\partial^2}{\partial t_1^2}$. By averaging equation (2) on time t_0 we obtain the following equation of pendulum's "slow" motion

$$\varepsilon^2\frac{d^2\alpha}{dt_1^2} - \delta(\alpha + \chi\langle\psi\cos t_0\rangle) = 0 \tag{4}$$

Equation of pendulum's "fast" motions may be obtained by subtracting equation (4) from equation (2)

$$\frac{\partial^2\psi}{\partial t_0^2} + 2\varepsilon\frac{\partial^2\psi}{\partial t_1\partial t_0} + \varepsilon^2\frac{\partial^2\psi}{\partial t_1^2} - \delta\psi = \delta\chi\left((\alpha+\psi)\cos t_0 - \langle\psi\cos t_0\rangle\right) \tag{5}$$

In conventional cases of the MDSM application [3] corresponding equation of fast motions is solved only approximately, because the equation of slow motion is the one of the primary interest. In particular, while solving this equation all involved slow variables are considered as constants ("frozen"), and terms $2\varepsilon\frac{\partial^2\psi}{\partial t_1\partial t_0}$ and $\varepsilon^2\frac{\partial^2\psi}{\partial t_1^2}$ are neglected, because they are small in comparison with $\frac{\partial^2\psi}{\partial t_0^2}$. In the present case this simplification has to be abandoned. Indeed, terms of order of ε^2 are retained in equation (4) of pendulum's slow motion and $\delta \sim 1$, $\chi \sim 1$, so solution of fast motions' equation should be found with the accuracy of order of ε^2. I.e. terms of this order, particularly $2\varepsilon\frac{\partial^2\psi}{\partial t_1\partial t_0}$ and $\varepsilon^2\frac{\partial^2\psi}{\partial t_1^2}$, should be retained in equation (5).

Taking into account that $\psi(t_1, t_0)$ is time t_0 periodic function, the solution of fast motions equation (5) is sought in the form of series

$$\psi = B_{11}(t_1)\cos t_0 + B_{12}(t_1)\sin t_0 + B_{21}(t_1)\cos 2t_0 + B_{22}(t_1)\sin 2t_0 + ... \tag{6}$$

Amplitudes $B_{11}(t_1)$, $B_{12}(t_1)$,... are determined by multiplying equation (5) sequentially by $\cos t_0$, $\sin t_0$,... and averaging on time t_0. Consequently, the following system of equations is derived

$$-B_{11} + 2\varepsilon\frac{dB_{12}}{dt_1} + \varepsilon^2\frac{d^2B_{11}}{dt_1^2} - \delta B_{11} = \delta\chi\left(\alpha + \frac{B_{21}}{2}\right) \tag{7}$$

$$-B_{12} - 2\varepsilon\frac{dB_{11}}{dt_1} + \varepsilon^2\frac{d^2B_{12}}{dt_1^2} - \delta B_{12} = \delta\chi\frac{B_{22}}{2} \tag{8}$$

...

$$-n^2B_{n1} + 2n\varepsilon\frac{dB_{n2}}{dt_1} + \varepsilon^2\frac{d^2B_{n1}}{dt_1^2} - \delta B_{n1} = \delta\chi\frac{B_{n-1,1}}{2} \tag{9}$$

$$-n^2B_{n2} - 2n\varepsilon\frac{dB_{n1}}{dt_1} + \varepsilon^2\frac{d^2B_{n2}}{dt_1^2} - \delta B_{n2} = \delta\chi\frac{B_{n-1,2}}{2} \tag{10}$$

A classical scheme of solution decomposition in orders of the small parameter is employed for solving this system of equations. Although in equations (7)-(10) small parameter is at highest derivative, in the considered case application of this scheme is justified. Indeed, at $O(\delta) = O(\chi) = 1$ due to the fact that amplitudes $B_{11}(t_1)$, $B_{12}(t_1)$,... should be "slow" functions, the solution of the corresponding homogeneous system turns out to be zero. So, the problem reduces to the determination of a particular solution of the inhomogeneous system (7)-(10). For this purpose the classical scheme of solution decomposition in orders of the small parameter may be employed. As the result we obtain:

$$B_{11}(t_1) = -F(\delta,\chi)\alpha(t_1) + \varepsilon^2 F_2(\delta,\chi)\frac{d^2\alpha}{dt_1^2} + O(\varepsilon^3), \tag{11}$$

$$B_{12}(t_1) = \varepsilon F_1(\delta,\chi)\frac{d\alpha}{dt_1} + O(\varepsilon^3), etc.$$

Here $F(\delta,\chi)$, $F_1(\delta,\chi)$, $F_2(\delta,\chi)$ are functions of parameters δ and χ, which depend significantly on the number of retained harmonics in series (6).

So, we have determined expressions for amplitudes $B_{11}(t_1)$, $B_{12}(t_1)$,..., which are correct with the accuracy of order of ε^2. Employing the solution of fast motions equation (5), the equation of slow motion is composed in the form

$$\varepsilon^2(1 - \delta\frac{\chi}{2}F_2(\delta,\chi))\frac{d^2\alpha}{dt_1^2} - \delta(1 - \frac{\chi}{2}F(\delta,\chi))\alpha = 0 \tag{12}$$

Taking into account that $t_1 = \varepsilon t_0$, this equation may be rewritten as

$$(1 - \delta\frac{\chi}{2}F_2(\delta,\chi))\frac{d^2\alpha}{dt_0^2} - \delta(1 - \frac{\chi}{2}F(\delta,\chi))\alpha = 0 \tag{13}$$

As it was noted above, functions $F(\delta,\chi)$, $F_2(\delta,\chi)$ depend significantly on the number of retained harmonics in series (6). While solving equation of fast motions, terms of order of ε^2 were taken into account, so number n of the harmonic, which may be discarded, is determined by the relation

$$\frac{1}{4}\frac{\delta\chi}{\delta + n^2}\frac{\delta\chi}{\delta + (n-1)^2} \sim \varepsilon^3 \tag{14}$$

Taking into account relations $\delta = \frac{mlg}{I\Omega^2}$, $\chi = \frac{G\Omega^2}{g}$ and $t_0 = \Omega t$, equation (13) may be rewritten in the form

$$\left[1 - \frac{mlG}{2I}F_2\left(\frac{mlg}{I\Omega^2}, \frac{G\Omega^2}{g}\right)\right]\frac{d^2\alpha}{dt^2} + \frac{mlg}{I}\left[\frac{G\Omega^2}{2g}F\left(\frac{mlg}{I\Omega^2}, \frac{G\Omega^2}{g}\right) - 1\right]\alpha = 0 \tag{15}$$

From equation (13) it follows that not only effective "stiffness" of the system changes due to the external loading, but also its effective "mass". This fact is especially remarkable. As is shown in the classical papers (see, e.g. [1,2]), in the studied system only the effective stiffness changes under the action of vibration. The distinction is due to consideration of the different ranges of the parameters. In [1,2] the case $\delta \sim \varepsilon^2 << 1$ is studied, when the frequency of external loading is much higher than the natural frequency of the pendulum in the absence of this loading, whereas in the present paper we consider the case $\delta \sim 1$. The effective natural frequency of the pendulum becomes equal to

$$\lambda = \sqrt{\frac{mlg}{I}\left[\frac{G\Omega^2}{2g}F\left(\frac{mlg}{I\Omega^2}, \frac{G\Omega^2}{g}\right) - 1\right]\Big/\left[1 - \frac{mlG}{2I}F_2\left(\frac{mlg}{I\Omega^2}, \frac{G\Omega^2}{g}\right)\right]} \tag{16}$$

It should be noted that obtained equation (13) of pendulum's slow motion is correct also at "conventional" values of the parameters $\delta \sim \varepsilon^2$ and $\chi \sim \frac{1}{\varepsilon}$. Indeed, in this case change of the system's effective "mass" $-\delta\frac{\chi}{2}F_2(\delta,\chi)$ is negligibly small. So, the results obtained in the present paper are in good agreement with the conclusions of the classical studies [1,2].

From obtained equation (13) it follows that the upper position of the pendulum becomes stable due to action of vibration, if the following condition holds true:

$$\frac{\frac{\chi}{2}F(\delta,\chi)-1}{1-\delta\frac{\chi}{2}F_2(\delta,\chi)}>0 \tag{17}$$

In the case $\delta\sim\varepsilon^2$, $\chi\sim\frac{1}{\varepsilon}$, taking into account relation $\delta\frac{\chi}{2}F_2(\delta,\chi)\sim\varepsilon^2$, this inequality may be reduced to the classical condition of the pendulum's upper position stabilization

$$\frac{\chi}{2}F(\delta,\chi)>1$$

3 On the validity of the results obtained by the proposed modification of the MDSM

In order to substantiate the proposed modification of the MDSM a correlation between this method and other approaches should be revealed. Consider the classical method of harmonic balance in Ritz's interpretation [11,12]. Application of this method also implies a hypothesis regarding the type of the solution sought. It is assumed that this solution is a time periodic function, i.e. that system in the considered range of parameters performs stationary oscillations. If this hypothesis is not correct, then the trivial solution or solution, which doesn't meet the sense of the problem, is obtained. Otherwise the parameters of oscillations of the considered type are determined. It should be noted that Ritz's method was validated in many classical papers (see, e.g. [11,12]).

So the main difference between the MDSM and the classical method of harmonic balance lies in hypothesis regarding the type of the solution sought. The MDSM may be interpreted as a method, by means of which not only stationary oscillations, but also oscillations with slowly varying characteristics may be determined. It should be noted that in this sense the MDSM is similar to the classical method of slowly varying amplitudes [12], which was proposed by Van der Pol for solving the equation named after him. However Van der Pol's method implies only the first harmonic in the solution being taken into account. Another approach, correlation with which should be mentioned, is the projection method [13]. This approach also implies the representation of solution of the considered equation as a series of orthogonal functions with coefficients, which vary slowly in comparison with these functions.

The MDSM requires an additional a posteriori analysis of the obtained results. It should be assayed, whether the characteristics of the defined oscillations vary indeed slowly in comparison with these oscillations, or not. In the considered problem, taking into account that all amplitudes $B_{11}(t_1)$, $B_{12}(t_1)$,... depend on variable $\alpha(t_1)$, this verification may be reduced to the assessment of the fulfillment of the following relation

$$\frac{d\alpha}{dt_0}\frac{1}{\alpha_A}<<\frac{1}{\psi_A}\frac{d\psi}{dt_0} \tag{18}$$

where α_A and ψ_A are characteristic amplitudes of variables $\alpha(t_1)$ and $\psi(t_1,t_0)$ oscillations. Taking into account equation (13), relation (18) may be reduced to the following one

$$\sqrt{\left|\delta\left[\frac{\chi}{2}F(\delta,\chi)-1\right]\Big/\left[1-\delta\frac{\chi}{2}F_2(\delta,\chi)\right]\right|} << 1 \tag{19}$$

Thereby, the obtained equation of pendulum's slow motion (13) and corresponding expressions for amplitudes $B_{11}(t_1)$, $B_{12}(t_1)$,... are correct, if condition (19) holds true. Taking into account equality (16), this condition may be written in traditional form $\lambda << \Omega$ [3,4].

It should be noted that application of the classical asymptotic methods, e.g. the MSM, also implies a posteriori analysis of the results obtained [7]. It is required, in particular, to assess a time interval, in which the defined solution is correct.

Taking into account that small parameter ε is present in decomposition (3), a correlation between the proposed modification of the MDSM and classical asymptotic methods may be revealed. As in the case of these methods, application of the modified MDSM requires an accurate account of orders of the parameters in the considered equation. This account is necessary to identify terms, which may be neglected while solving corresponding equations of fast and slow motions. In the considered case, when $\delta \sim 1$ and $\chi \sim 1$, solution of the fast motions equation (5) was found with the accuracy of order of ε^2 in order to compose correct equation of slow motion (4). It should be noted that in the classical case $\delta \sim \varepsilon^2$ and $\chi \sim \frac{1}{\varepsilon}$ this solution may be determined with the accuracy of order of ε^0.

At the same time, employment of the MDSM doesn't require the presence of a small parameter in initial equation. This distinction broadens significantly its applicability range in comparison with the classical asymptotic methods.

We note that the modification of the MDSM, proposed in the present paper, may be considered as a generalization of the procedure of this method application, which was employed in [15] for the analysis of equations with high-frequency modulations of dissipation coefficients.

4 Comparison with the results of numerical experiments

In accordance with the results obtained in section 2 in the considered range of parameters not only the effective "stiffness" of the system changes due to the external loading, but also its effective "mass". Corresponding condition of pendulum's upper position stabilization with expressions (17), (19) being taken into account may be written in the form

$$0 < \frac{\frac{\chi}{2}F(\delta,\chi)-1}{1-\delta\frac{\chi}{2}F_2(\delta,\chi)} << 1 \tag{20}$$

A series of numerical experiments was conducted to verify these results. Initial equation (2) was integrated directly by means of the Wolfram Mathematica 7; corresponding results were compared with the solution of equation (13) of pendulum's slow motion.

Consider the case $\delta = \frac{mlg}{I\Omega^2} = 0.4$ as an illustrative example. For such δ and $\chi \sim 1$ condition (14) fulfils for $n = 4$, so only three harmonics may be taken into account in solution (6) of pendulum's fast motions equation. Condition (20) of pendulum's upper position stabilization in this case reduces to $0 < \chi - 2.587 << 1$. The dependence of pendulum's deflection φ on time t_0 at $\delta = 0.4$, $\chi = 2.59$ is shown in Figure 1(a) for initial conditions $\varphi(0) = 0.1$, $\dot{\varphi}(0) = 0$; in Figure 1(b) this dependence is presented for $\chi = 2.585$ and the same values of other parameters. Solid line is the numerical solution of the initial

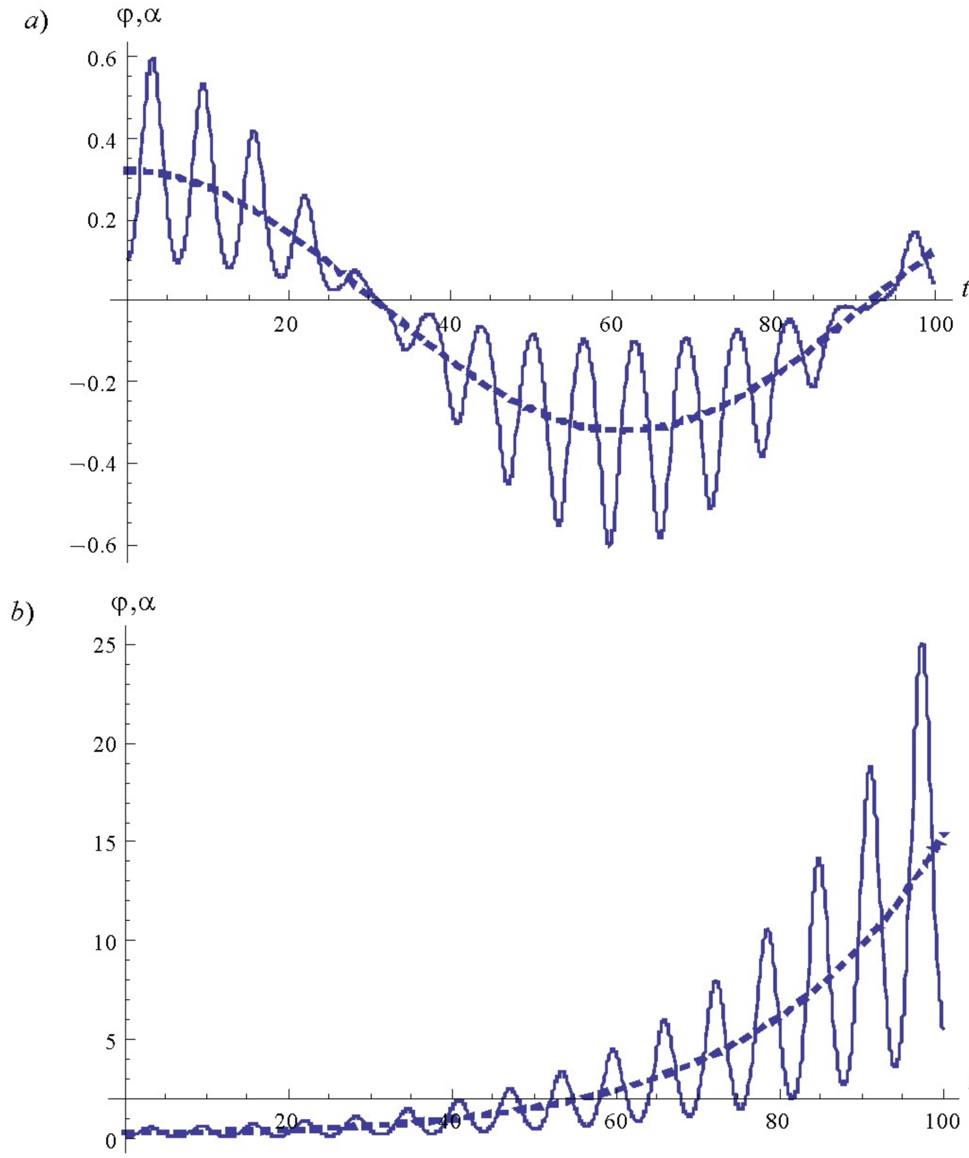

Figure 1: The dependence of pendulum deviation angle φ on time t_0 at $\delta = 0.4$ and a) $\chi = 2.59$, b) $\chi = 2.585$; initial conditions $\varphi(0) = 0.1$, $\dot{\varphi}(0) = 0$. Solid line is the numerical solution of the initial equation (2), dashed line is solution $\alpha(t_0)$ of the obtained equation of pendulum's slow motion (13).

equation (2), dashed line is solution $\alpha(t_0)$ of the obtained equation of pendulum's slow motion (13).

As another illustrative example consider the case $\delta = 1.4$. For such δ condition (14) fulfils only for $n = 6$, so five harmonics were taken into account while solving equation (5) of pendulum's fast motions. Condition (20) of pendulum's upper position stabilization in this case reduces to $0 < \chi - 1.73944 << 1$. The dependencies of pendulum's deviation angle on time t_0 at $\delta = 1.4$ and a) $\chi = 1.7395$, b) $\chi = 1.7394$ are shown in Figure 2 for initial conditions $\varphi(0) = 0.01$, $\dot{\varphi}(0) = 0$.

As is seen from Figures 1-2, analytical solution is in good agreement with the results of numerical experiments. In particular, our conclusion that not only the effective "stiffness"

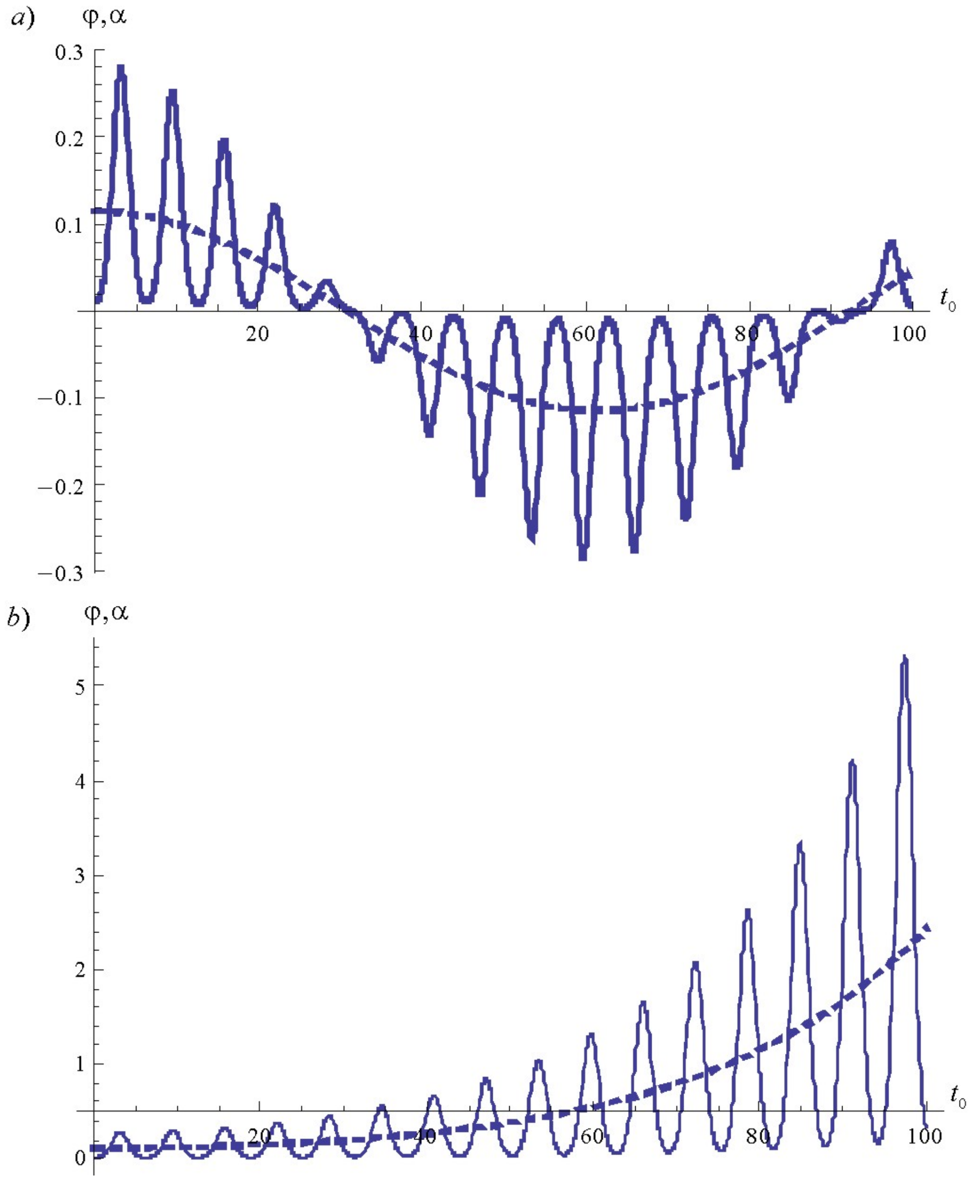

Figure 2: The dependence of pendulum deviation angle on time t_0 at $\delta = 1.4$ and a) $\chi = 1.7395$, b) $\chi = 1.7394$; initial conditions $\varphi(0) = 0.01$, $\dot{\varphi}(0) = 0$. Solid line is the numerical solution of the initial equation (2), dashed line is solution $\alpha(t_0)$ of the obtained equation of pendulum's slow motion (13).

of the system changes due to the external loading, but also its effective "mass", is confirmed. Also the obtained condition of pendulum's upper position stabilization (20) is validated. We note that validity of this condition is confirmed also by the Ince-Strutt diagram [11,12].

5 Solution by means of the MSM

Employ the MSM for solving equation (2) in the case $O(\delta) = O(\chi) = 1$. Application of the MSM and other classical asymptotic methods implies the presence of a small parameter in the equation under study. In the considered case it is impossible to assign such parameter in equation (2), since $O(\delta) = O(\chi) = 1$ and $\delta \not\approx 1$. So, small parameter ε should be introduced artificially in this equation in order to apply asymptotic methods, e.g. in the following manner:

$$\frac{d^2\varphi}{dt_0^2} - \varepsilon\delta(\varepsilon + \chi\cos t_0)\varphi = 0 \tag{21}$$

This manner of introducing the small parameter corresponds to the classical case, studied, e.g. in papers [4,7]. Following the MSM solution of equation (21) is sought in the form of series

$$\varphi = \varphi_0(t_0, t_1) + \varepsilon\varphi_1(t_0, t_1) + \varepsilon^2\varphi_2(t_0, t_1) + ... \tag{22}$$

where φ_0, φ_1, φ_2 are 2π - periodic functions of t_0.

As the result for the main component φ_0 of pendulum oscillations we obtain the following equation

$$\frac{d^2\varphi_0}{dt_0^2} + \delta\left(\frac{1}{2}\delta\chi^2 - 1\right)\varphi_0 = 0 \tag{23}$$

As is seen, this equation differs substantially from the correct equation (13) obtained by means of the MDSM. Particularly, it doesn't detect the revealed effect of changing of the effective "mass" of the system under the action of vibration. Moreover, value of the effective "stiffness" of the system, determined by this equation, differs from the correct value $\delta(\frac{\chi}{2}F(\delta,\chi) - 1)$. E.g. even if we take into account only the first harmonic in solution (6) of the fast motions equation, then expression $F(\delta,\chi) = \frac{\delta\chi}{1+\delta} \not\approx \delta\chi$ will be obtained for function $F(\delta,\chi)$.

Thereby, since stabilization of pendulum's upper position in the considered case may be achieved only in relatively narrow range of parameters and $\delta \sim 1$, we may conclude that stability condition $\frac{1}{2}\delta\chi^2 > 1$, obtained by means of the MSM, is not correct. It should be noted that equation (23) can not be refined by taking into account additional time scales in solution (22) $t_2 = \varepsilon^2 t_0$, $t_3 = \varepsilon^3 t_0$ etc.

Consider other manners of small parameter ε introducing in the initial equation (2), e.g. the following one

$$\frac{d^2\varphi}{dt_0^2} - \varepsilon^2\delta(1 + \chi\cos t_0)\varphi = 0 \tag{24}$$

In this case resulting equation for the main component φ_0 will take the form

$$\frac{d^2\varphi_0}{dt_0^2} - \delta\varphi_0 = 0 \tag{25}$$

As is seen, this equation doesn't reflect the change of the system's effective parameters under the action of external loading. Introducing small parameter ε in the following ways

$$\frac{d^2\varphi}{dt_0^2} - \delta(1 + \varepsilon\chi\cos t_0)\varphi = 0 \tag{26}$$

$$\frac{d^2\varphi}{dt_0^2} - \varepsilon\delta(1 + \chi\cos t_0)\varphi = 0 \tag{27}$$

$$\frac{d^2\varphi}{dt_0^2} - \varepsilon\delta(1 + \varepsilon\chi\cos t_0)\varphi = 0 \tag{28}$$

we obtain $\varphi_0 = 0$, since functions φ_0 and φ_1 should be periodic in time t_0.

In the case $\delta \approx 1$ small parameter $(\delta - 1)$ is present in equation (2) in explicit form. However, a correct condition of pendulum's upper position stabilization also can't be obtained in this case by means of the MSM.

Thus, application of the MSM for solving equation (2) in the considered range of parameters leads to erroneous results. So, the proposed modification of the MDSM may be employed in cases, when classical asymptotic methods can not be used, i.e. the applicability range of the MDSM is broader than the one of the mentioned methods.

6 Conclusions

A modification of the MDSM applicable for solving equations, which don't contain a small parameter explicitly, is proposed in the paper. As an illustrative example a classical problem about the stability of a pendulum with vibrating suspension axis is considered in the case, when the frequency of external loading and the natural frequency of the pendulum in the absence of this loading are of the same order. As the result, a condition of pendulum's upper position stabilization is determined for this case. It is noted that in the considered range of parameters not only the effective "stiffness" of the system changes due to the external loading, but also its effective "mass". This fact is especially remarkable, since in the classical case, when the frequency of external loading is much higher than the natural frequency of the pendulum, only the effective stiffness of the system changes.

The validity of the results obtained by the proposed modification of the MDSM is confirmed. A correlation between the MDSM and Ritz's method of harmonic balance, Van der Pol's method of slowly varying amplitudes, the classical asymptotic methods and other approaches is revealed. It is shown that application of the classical asymptotic methods for solving the considered equation in the studied range of parameters leads to erroneous results. So, the applicability range of the MDSM turns out to be broader than the one of these methods.

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