Permanent magnetic levitation of Levitron using periodic magnetic forcing

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Abstract

Levitron is a toy that consists in a spinning top that levitates over a magnetic base. In the original toy the air drag decreases the spin rate until, after two or three minutes, the top destabilizes and falls. It is possible to achieve levitation during long periods of time driving the top with an alternating magnetic field. We report measurements of stable levitation for the latter case and analyze theoretical and experimentally the top dynamics. It is demonstrated that the magnetic torque that drives the top is due to a misalignment between the magnetic dipolar moment and the mechanical axis of the top.

1 Introduction

Levitron is a toy that consists on a magnetic top that hovers freely above a magnetic base. The top levitates due to the magnetic repulsion between the top and the base. The gyroscopic effect helps to maintain the axis of the top against flipping.

The dynamics of Levitrons has been studied by several authors [1–8]. Energetic considerations impose some restrictions on the design of the magnet of the base. Additionally some of these authors analyzed the dynamical conditions for a stable levitation. There are lower and upper limits for the spinning velocity of the top. Outside these limits the levitation is not stable.

In the commercial Levitron the levitation last a time of the order of two minutes, because the air friction slows down the spinning and the angular velocity eventually reaches the lower stability limit and falls down. The company that commercializes Levitron also commercializes *Perpetuator*, a device aimed to compensate the friction losses, producing a much longer levitation. *Perpetuator* is based in a US patent [9]: an alternating magnetic field produces the required torque. Simon *et al.* [2] used a similar device in their studies to control the top spin velocity.

In this paper we present some experimental, theoretical and numerical results that help to understand how an AC magnetic field help to sustain levitation. We have used a pair of Helmholtz coils to produce the alternating field. We write the equations of motion, including the AC external magnetic field and the air friction. An approximate solution is found, for the case of small precession angle, that helps to understand the basic forcing mechanism. Finally the whole set of equations are solved numerically, using Matlab. We show that, in order to power up the spinning, the magnetic dipole of the top must not be exactly aligned with the symmetry axis. This deviation was already proposed by Flanders *et al.* as an explanation of certain dynamical features of the undriven top [7].

2 Experimental observations

The experimental setup is depicted in Figure 1. The base is placed at the center of two Helmholtz coils that are driven by a function generator. A fast camera allows to record the top motion for later analysis. We have measured the mechanical and magnetic characteristics of the top. Its weight is 13.70 g. The washers supplied with Levitron may increase this weight up to 23.4 g. The diameter of the top is 29.5 mm and the height of the ceramic part is 5.3 mm. The corresponding moments of inertia are $I_1 = 1.2 \times 10^{-6}$ kg m^2 and $I_3 = 2.3 \times 10^{-6}$ kg m^2, where I_3 is the moment of inertia about the axis of symmetry and I_1 is the moment of inertia about an axis perpendicular to the axis of symmetry. They have been computed taking 21.2 g as the mass of the top. Measuring the magnetic field produced by the top we have established its magnetic dipolar moment $\mu = 0.91\,\mathrm{Am}^2$.

When the top is hovering freely, without external driving, the precession and spin rates are almost the same. This coupling between precession and spinning was observed by Flanders *et al.*[7], and explained by them as a consequence of a small angle Δ of deviation between the magnetic moment and the top axis. We have measured this angle for various tops and they are in the range 1 to 4 degrees. The measurements that we present in this paper correspond to a top of $\Delta = 2.9^o = 0.05$ rad.

The typical time of levitation without forcing is around two minutes. With the help of the Helmholtz coils, stable levitation can be achieved for periods of time much longer than two minutes. In some cases the levitation lasted for 2 or 3 days. It is not easy to obtain long stable levitations. It is more difficult to lift the top in the presence of the field of the Helmholtz coils than without it. Also many essays lead to a very unstable motion that end in the top falling down after some minutes. Only a small fraction of the tries lead to long levitations. All the points shown in Figure 2 correspond to levitations longer than 10 minutes, when the features of the dynamics were well established. This was possible for AC frequencies in the range 10 to 50 Hz.

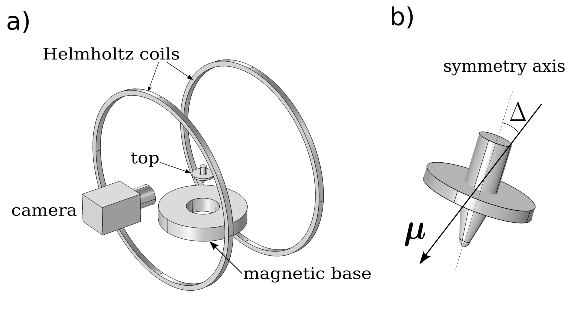

Figure 1: Experimental setup.

The fast camera recordings clearly show that the velocity of precession follows the external AC field. Figure 2 shows measurements of the top precession frequency as a function of the frequency of the alternating magnetic field. The amplitude of the alternating field was 0.49 mT. The data are the result of averaging over, at least, five precession periods. It is clearly observed that the precession and the alternating field frequencies coincide within all the range, indicating that the top precession couples to the torque exerted by the alternating field on it.

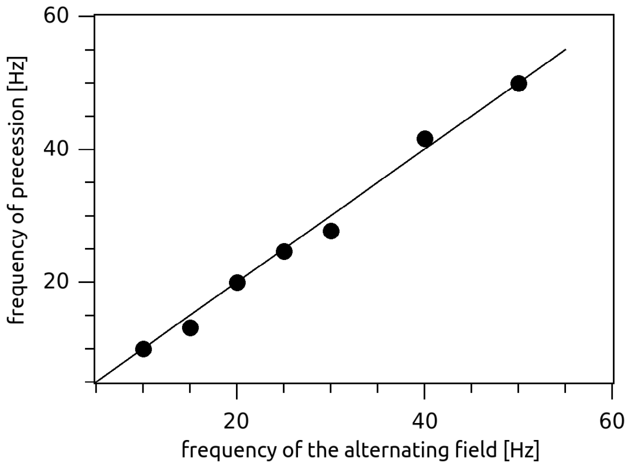

Figure 2: Precession frequency of the top as a function of the frequency of the alternating magnetic field (magnetic field amplitude 0.49 mT).

Along with this synchronous precession we observe also the synchronization of the spin of the top around itself with the precession, something already found in the absence of external driving.

3 Mathematical model

The magnetic field created by the magnetic base near the levitation point can be expressed as [4]:

$$B_x = \frac{1}{2}xV''(z) \quad (1)$$

$$B_y = \frac{1}{2}yV''(z) \quad (2)$$

$$B_z = -V'(z) + \frac{1}{4}(x^2+y^2)V'''(z) \quad (3)$$

Following Dullin *et al.*, we chose the magnetic scalar potential at the axis $V_0(z)$ as the one due to a disk of radius a with a hole of radius b:

$$V(z) = 2B_0\pi z\left(\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{a^2+z^2}}\right) \quad (4)$$

We measured the magnetic field produced by the base with the help of a teslameter. A non-linear fit allows us to obtain the parameters $a = 5.4$ cm, $b/a = 0.35$ and $B_0 = 7.6$ mT, that characterize the field.

Assuming that $B_z > 0$, the conditions of stability imply [1, 2, 3]

$$V''(z_c) > 0 \quad (5)$$

$$V'''(z_c) < 0 \quad (6)$$

$$V'''(z_c) - \frac{1}{2}\frac{(V''(z_c))^2}{V'(z_c)} > 0 \quad (7)$$

where z_c is the coordinate of the point of levitation. The first condition is required for the force between the base and the top to be repulsive. For $b/a = 0.35$, the last two conditions imply:

$$z_c/a > 1.057 \tag{8}$$
$$z_c/a < 1.134 \tag{9}$$

The trapping region, i.e. where these conditions are met and stable levitation is achieved, is rather small, less than half a centimeter. This is one of the circumstances that makes difficult to master the toy.

The dynamics of the top is described by the three Cartesian coordinates, x, y, and z of its center of mass, and the three Euler angles that give the spatial orientation.

The Lagrangian function associated to the top motion is given by

$$\begin{aligned}\mathcal{L}(x,y,z,\psi,\theta,\phi) &= \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)\\ &+\frac{1}{2}I_1(\dot{\theta}^2+\sin^2\theta\dot{\phi}^2)+\frac{1}{2}I_3(\dot{\psi}+\cos\theta\dot{\phi})^2-U(x,y,z,\psi,\theta,\phi)\end{aligned} \tag{10}$$

where m is the top mass.

The Helmholtz coils couple to the residual transverse magnetization of the top. In order to be able to describe the effect of this transverse magnetization, we assume that the axis of magnetization makes an angle Δ with respect to the mechanical axis of symmetry (Figure 1B)). Taking the $x-z$ plane as the plane that contains the magnetic dipole in the frame comoving with the top, the magnetic dipole moment may be written as:

$$\boldsymbol{\mu} = -(\cos\Delta\mathbf{n}_z + \sin\Delta\mathbf{n}_x)\mu \tag{11}$$

where μ is the magnitude of the dipole. The minus sign is to indicate that the dipole moment is pointing downwards, a condition required for equilibrium if we assume that the base magnetic field is pointing upwards. The unitary vectors $\mathbf{n}_x$ and $\mathbf{n}_z$ are:

$$\begin{aligned}\mathbf{n}_x &= (\cos\psi\cos\phi-\cos\theta\sin\phi\sin\psi)\mathbf{e}_x\\ &\quad -(\cos\psi\sin\phi+\sin\psi\cos\phi\cos\theta)\mathbf{e}_y\\ &\quad +\sin\psi\sin\theta\mathbf{e}_z \qquad (12)\\ \mathbf{n}_z &= \sin\theta\sin\phi\mathbf{e}_x-\sin\theta\cos\phi\mathbf{e}_y+\cos\theta\mathbf{e}_z. \qquad (13)\end{aligned}$$

Here $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the Cartesian unitary vectors referred to the laboratory frame of reference.

The potential energy becomes

$$\begin{aligned}U &= mgz+\mu\cos\Delta(B_x\sin\theta\sin\phi-B_y\sin\theta\cos\phi+B_z\cos\theta)\\ &\quad +\mu\sin\Delta(B_x(\cos\psi\cos\phi-\cos\theta\sin\phi\sin\psi)\\ &\quad +B_y(\cos\psi\sin\phi+\sin\psi\cos\phi\cos\theta)\\ &\quad +B_z\sin\psi\sin\theta)\end{aligned} \tag{14}$$

The Helmholtz coils produced a time dependent uniform magnetic field that can be expressed as:

$$\delta\mathbf{B} = \epsilon B_0\cos(\omega t)\mathbf{e}_y \tag{15}$$

where we have taken the y axis as the axis of the Helmholtz coils.

The energy losses come from the air friction. Considering only the friction due to the angular velocity of the top about its axis of symmetry, we introduce the auxiliary function:

$$\mathcal{F} = \frac{1}{2}\nu_3(\dot{\psi} + \cos\theta\dot{\phi})^2 \tag{16}$$

The equations of motion are the Euler-Lagrange equations, including an additional friction term:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = -\frac{\partial \mathcal{F}}{\partial \dot{q}_i} \tag{17}$$

where q_i represents any of the six variables x, y, z, ψ, θ, and ϕ.

The above set of equations have been analyzed by several authors in the absence of external varying field, without friction and without misalignment between the magnetic dipole and the top axis. The limits of stable levitation were computed by [4, 2, 5, 6, 8]. Flanders et al. [7] demonstrated that the inclusion of a small misalignment Δ produces the synchronization between proper rotation and precession.

3.1 Approximate solution for $\theta \sim 0$ and a uniform magnetic field

The equations of motion (17) do not admit analytical solution. However, an overall comprehension of the effect of the external forcing can be obtained in a simplified version of them. Let us consider the equation (17) that corresponds to $q_i = \psi$:

$$\begin{aligned} &I_3(\ddot{\psi} + \cos\theta\ddot{\phi}) - I_3\dot{\phi}\dot{\theta}\sin\theta = \\ &\mu B_z \sin\Delta \big(\cos\psi\sin\theta + \epsilon\cos(\omega t))(\cos\psi\cos\phi\cos\theta - \sin\psi\sin\phi)\big) \\ &-\nu_3(\dot{\psi} + \cos\theta\dot{\phi}) \end{aligned} \tag{18}$$

This is the dynamical equation of the rotation of the top about its axis. An inspection of this equation reveals that when $\Delta = 0$ the external AC field does not exert any torque on the axis of the top. That is, there is no way of driving externally the top if the magnetic moment is exactly aligned with its axis.

This equation is greatly simplified in the limit $\theta = 0$:

$$\begin{aligned} &I_3(\ddot{\psi} + \ddot{\phi}) = \\ &\mu B_z \sin\Delta\epsilon\cos(\omega t)(\cos\psi\cos\phi - \sin\psi\sin\phi) - \nu_3(\dot{\psi} + \dot{\phi}) \end{aligned} \tag{19}$$

or, introducing $\alpha = \psi + \phi$:

$$I_3(\ddot{\alpha}) = \mu B_z \sin\Delta\epsilon\cos(\omega t)\cos\alpha - \nu_3\dot{\alpha} \tag{20}$$

Exactly the same equation is obtained from (17) applied to the angle ϕ in the same limit.

We stress that the solutions to this equation do not represent exact solutions of the complete set of equations. This is due to the fact that there is no solutions with $\theta = 0$ when $\Delta \neq 0$, because it appears a torque $\mu B_z \sin\Delta\sin\psi\cos\theta$ that tends to flip the top from its axis. However, under certain conditions the angle θ remains small, and equation (20) helps to understand the dynamics.

Equation (20) is of the form:

$$\ddot{\alpha} + \gamma\dot{\alpha} - \beta\cos(\omega t)\cos\alpha = 0 \tag{21}$$

with $\gamma = \nu_3/I_3$ and $\beta = \mu B_z\epsilon\sin\Delta/I_3$. It resembles the equation of motion of a parametric pendulum. This equation does not posses steady solutions. Since the magnetic torque goes as $\cos(\omega t)\cos\alpha$, the only way to obtain a non-zero average torque that compensates the friction losses is to have $\alpha \sim \omega t$. That is, permanent levitation is possible only if the rotation of the top is synchronized with the AC magnetic field.

We have two kinds of solutions: small oscillations around $\alpha = 0$, with a zero average torque, and small oscillations around $\alpha = \omega t$. In both cases the oscillations will occur with a typical period much longer than $2\pi/\omega$. Let us introduce a new variable $r(t)$[10], where the dependence in t is very slow compare to ωt:

$$r(t) = \alpha - \alpha_0 - \omega t \tag{22}$$

At zero order in r and taking the average over t, equation (21) gives:

$$\cos\alpha_0 = \frac{2\gamma\omega}{\beta} \tag{23}$$

At first order in r we get:

$$\ddot{r} + \gamma\dot{r} + \frac{1}{2}\beta\sin\alpha_0 r = 0 \tag{24}$$

The solutions of this equation are of the form $r = r_0 e^{\lambda t}$ with

$$\lambda = -\frac{\gamma}{2} \pm i\sqrt{\frac{1}{2}\beta\sin\alpha_0 - \frac{1}{4}\gamma^2} \tag{25}$$

Therefore, the *equilibrium* $\cos\alpha_0 = 2\gamma\omega/\beta$ is stable if $\sin\alpha_0 > \gamma^2/(2\beta)$. In our case $\gamma\omega/\beta << 1$ which gives $\alpha_0 \sim \pi/2$ and the equilibrium, provided it exists, is always stable.

The function $r(t)$ features oscillations of decreasing amplitude of frequency

$$\omega_r = \sqrt{\frac{1}{2}\beta\sin\alpha_0 - \frac{1}{4}\gamma^2} \simeq \sqrt{\frac{1}{2}\beta\sin\alpha_0} \simeq \sqrt{\frac{1}{2}\beta} \tag{26}$$

A numerical study of equation (21) confirms all these predictions. For comparison to the case of the parametric pendulum, we investigated the basin of attraction of the non-zero solution. Figure 5 shows a plot of the initial conditions (in α and $\dot{\alpha}$) that results in a mean angular velocity ω. It is noted that not only values close to ω result in an efficient driving. A conspicuous feature of this diagram is that there are points that lead to $\dot{\alpha} \sim \omega$ very close to points that lead to $\dot{\alpha} \sim 0$. This fact is consistent with the difficulties encountered in flying the top.

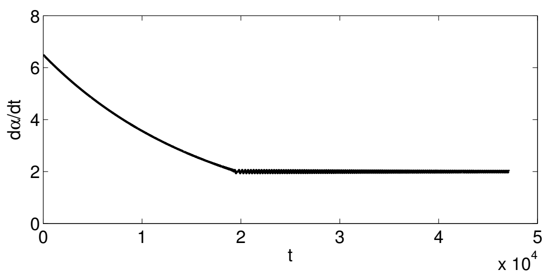

Figure 3: Evolution of $\dot{\alpha}$ for equation (20). The parameters are: $\beta = 2.5 \times 10^{-3}$, $\gamma = 6 \times 10^{-5}$ and $\omega = 2$. Initial conditions are $\alpha_0 = 0.38\pi$ and $\dot{\alpha}_0 = 6.5$.

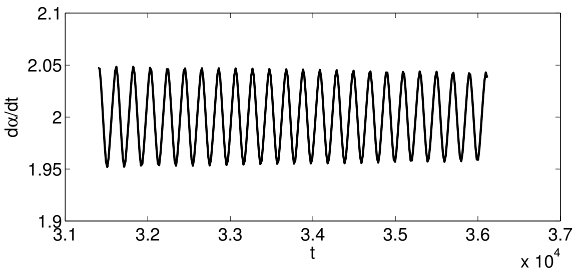

Figure 4: Detail of the previous figure.

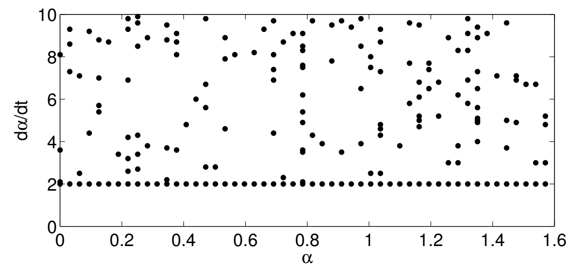

Figure 5: Basin of attraction of the solution with $\dot{\alpha} = \omega$ for equation (20). The parameters are: $\beta = 2.5 \times 10^{-3}$, $\gamma = 6 \times 10^{-5}$ and $\omega = 2$. The initial conditions cover the rectangle with a mesh of $\Delta\alpha_0 = 0.01\pi$ and $\Delta\dot{\alpha}_0 = 0.1$.

4 Numerical simulations

A system of this complexity can be studied in detail only with the help of numerical solutions. The classical definition of Euler angles leads to a system of equations that is singular when the top is spinning around its axis of symmetry. In order to avoid this singularity we use the yaw-pitch-roll sequence. The top orientation is given by three angles (ψ, θ and ϕ) that represents successive rotations about the x, y, and z axis. In our definition the rotations abour the x and z axis are clockwise, and the rotation about the y axis is counterclockwise. The coordinates of any point of the top with respect to a fixed frame of reference can be found by applying the matrix of rotation $R(\psi, \theta, \phi)$ to the coordinates of the point in a frame comoving with the top. The details can be found in [11].

For the numerical integration is convenient to define non-dimensional magnitudes in order to deal with numbers of the order of the unity. We are going to take $\tau = \sqrt{I_1/(\mu B_0)}$ as a scale for time and the parameter a, the radius of the magnet of the base, as a scale for distances. In order to get a first order system of equations it is preferred to use a Hamiltonian formulation.

Following Dulling et al.[4], the Hamiltonian function becomes :

$$\mathcal{H}(x, y, z, p_x, p_y, p_z, \psi, \theta, \phi) = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2)$$
$$+\frac{1}{2\lambda}(p_\theta^2 + \frac{(p_\phi - p_\psi \sin\theta)^2}{\cos^2\theta}) + \frac{p_\psi^2}{2} + U(x, y, z, \psi, \theta, \phi) \qquad (27)$$

Hamilton's equation are:

$$\dot{x} = p_x, \quad \dot{y} = p_y, \quad \dot{z} = p_z, \qquad (28)$$

$$\dot{p}_x = -\frac{\partial U}{\partial x}, \quad \dot{p}_y = -\frac{\partial U}{\partial y}, \quad \dot{p}_z = -\frac{\partial U}{\partial z} \tag{29}$$

$$\dot{\psi} = \lambda p_\psi + \frac{\sin\theta}{\cos^2\theta}(p_\psi \sin\theta - p_\phi), \tag{30}$$

$$\dot{\theta} = p_\theta, \tag{31}$$

$$\dot{\phi} = -\frac{1}{\cos^2\theta}(p_\psi \sin\theta - p_\phi), \tag{32}$$

$$\dot{p}_\psi = -\frac{\partial U}{\partial \psi} - \nu p_\psi, \tag{33}$$

$$\dot{p}_\theta = \frac{1}{\cos^3\theta}(p_\psi \sin\theta - p_\phi)(p_\phi \sin\theta - p_\psi) - \frac{\partial U}{\partial \theta}, \tag{34}$$

$$\dot{p}_\phi = -\frac{\partial U}{\partial \phi}. \tag{35}$$

where the potential energy U is:

$$\begin{aligned} U \;=\;& Gz + \frac{1}{\Lambda}\cos\Delta(B_x \sin\theta + B_y \cos\theta \sin\phi + B_z \cos\theta\cos\phi) \\ & +\mu \sin\Delta(B_x \cos\psi\cos\theta - B_y(\cos\phi\sin\psi + \cos\psi\sin\phi\sin\theta) \\ & +B_z(\sin\phi\sin\psi - \cos\phi\cos\psi\sin\theta)) \end{aligned} \tag{36}$$

and the non dimensional magnetic field is given by (3) with $V(z)$ the non-dimensional form of the potential:

$$V(z) = 2\pi z \left(\frac{1}{\sqrt{b^2 + z^2}} - \frac{1}{\sqrt{1 + z^2}} \right) \tag{37}$$

The y component of the magnetic field includes the time-dependent term. Also a friction term has been added to the right of equation (33)

Three non-dimensional parameters appear in these equations:

$$\Lambda = \frac{ma^2}{I_1} \quad G = \frac{gI_1}{a\mu B_0} \quad \lambda = \frac{I_1}{I_3} \tag{38}$$

We used the function ODE45, the standard Matlab solver for systems of ordinary differential equations, to solve the system of equations (28)-(35). The code was tested solving different cases for $\Delta = 0$, without forcing and without friction, and comparing with the available literature. The details can be found in [11].

Figure 6 shows the trajectory of the top in the $x - z$ plane for a complete simulation, when friction and forcing are taking into account. The center of mass of the top explores a significant part of the region where stable levitation is possible. The trajectory is rather wavy, a fact that is also observed experimentally.

The forcing makes permanent levitation possible. But the magnetic torque can compensate the friction losses only if the rotation of the top around itself is synchronized with the external field. Figure 7 shows the angular velocity Ω_3 around the axis of the top as a function of time for a given set of parameters. The non-dimensional angular frequency of the external field is 2. The angular velocity of the top oscillates around this value, and, therefore, is synchronize on average. The oscillations have a period of 244, much longer than the velocity of rotation. This corresponds to an angular frequency of 0.026, to be compared with the prediction of the approximation given by equation (26), which gives 0.027. (Note: this frequency is non-dimensional, therefore it is necessary to multiply the factor in (26) by $\sqrt{I_1/(\mu B_0)}$. In addition, the magnetic field is affected by a factor $V'(z_c) = 1.42$.)

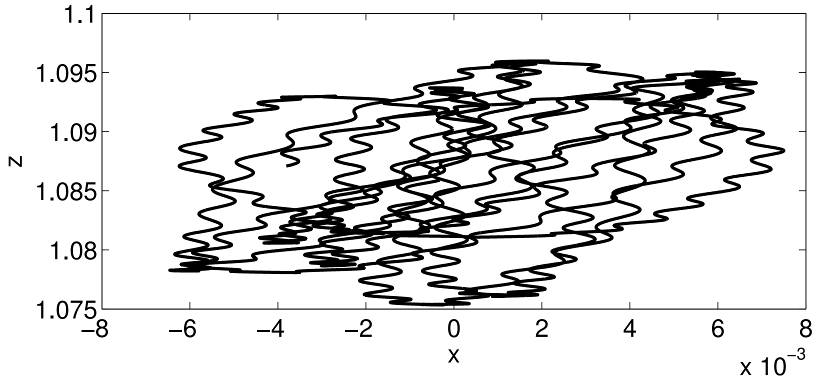

Figure 6: Trajectory of the top in the (x, z) plane. The non-dimensional parameters are $\lambda = 0.52$, $b = 0.35$, $\Lambda = 51.3$, $G = 0.0315$, $\Delta = 0.05$, $\nu = 5.74 \times 10^{-5}$, and $\epsilon = 0.04$. The initial angular velocity is $\dot{\psi}_0 = 2$.

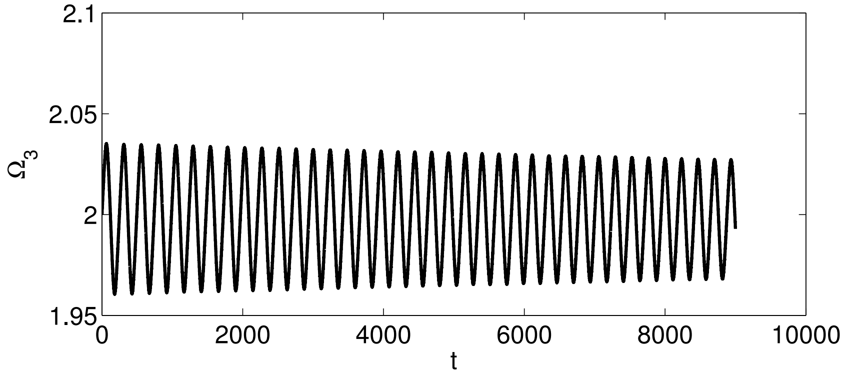

Figure 7: Angular velocity of the top as a function of time when an AC transversal magnetic field is applied. The parameters are $\lambda = 0.52$, $b = 0.35$, $\Lambda = 51.3$, $G = 0.0315$, $\Delta = 0.05$, $\nu = 5.74 \times 10^{-5}$, $\epsilon = 0.04$ and $\omega_c = 2$. The initial angular velocity is $\dot{\psi}_0 = 2$.

As we observed experimentally, the precession is also synchronized with the magnetic field. This is shown in Figure 8, where the supplementary of the angle of precession is plotted along with the phase of the external field. When boht line coincides, the torque takes its maximum value. The plot shows that the precession follows the field phase on average, with some small fluctuations.

5 Conclusion

Our experiments show that long stable levitation of a Levitron can be achieved with the help of a horizontal alternating magnetic field. The top precession synchronizes with the proper rotation and couples to the frequency of the alternating field. The magnetic torque compensates the air friction on the top. A simplification of the equations for the top dynamics allows to clarify the physical mechanism that leads to synchronization. A misalignment between the magnetic and the mechanical axis of symmetry is needed in order to achieve permanent levitation. In addition, this simplify equation predicts correctly the frequency of the small oscillation of the top spin velocity around the frequency of the

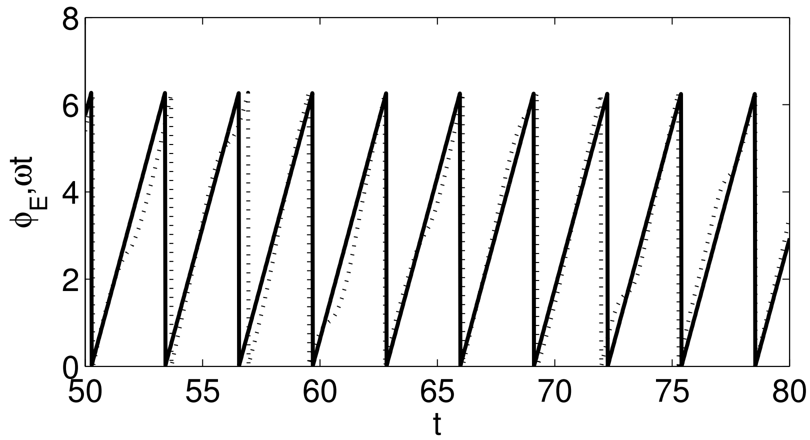

Figure 8: Supplementary of the angle of precession (solid line) and phase of the external magnetic field (dotted line) as a function of time. When the angle and the phase are the same, the magnetic torque is maximum. The parameters are $\lambda = 0.52$, $b = 0.35$, $\Lambda = 51.3$, $G = 0.0315$, $\Delta = 0.05$, $\nu = 5.74 \times 10^{-5}$, and $\epsilon = 0.04$. The initial angular velocity is $\dot{\psi}_0 = 2$.

magnetic field. We also conducted numerical simulations of the whole set of equations that illustrate the details of the top dynamics.

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References

[1] M. V. Berry "The Levitron (TM): an Adiabatic Trap for Spins", Proc. R. Soc. London, Ser. A **452**, 1207B–1220 (1996).

[2] Martin D. Simon, Lee O. Helfinger and S. L. Ridgway, "Spin stabilized magnetic levitation", Am. J. Phys. vol. **65** (4), 286–292 (1997).

[3] T. B. Jones, Masao Washizu and Roger Gans "Simple theory for the Levitron", J. Appl. Phys. **82**, 883–888 (1997).

[4] Holger R. Dullin and Robert W. Easton, "Stability of Levitrons", Physica D **126**, 1–17 (1999).

[5] G. Genta, C. Delprete and D. Rondano, "Gyroscopic Stabilization of Passive Magnetic Levitation", Meccanica **34**, 411–424 (1999).

[6] S. Gov, S. Shtrikman and H. Thomas, "On the dynamical stability of the hovering magnetic top", Physica D **126**, 214-224 (1999).

[7] P. Flanders, S. Gov, S. Shtrikman and H. Thomas, "On the spinning motion of the hovering magnetic top" Physica D **126**, 225-235 (1999)

[8] A. San Miguel, "Numerical integration for the dynamics of the heavy magnetic top" Physics Letters A **335**, 235-244 (2005)

[9] Hones et al."Electromagnetic drive method and apparatus for driving a rotationally stabilized magnetic levitated object". Patent Nr: **5,883,454**, (1999).

[10] B. Horton, J. Sieber, J.M.T. Thompson and M. Wiercigroch "Dynamics of the nearly parametric pendulum" International Journal of Non-Linear Mechanics **46**, 436-442 (2011).

[11] A. T. Pérez and Pablo García Sánchez "Dynamics of a Levitron under a periodic magnetic forcing". Submitted to Am. J. Phys.

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