Nonlinear Vibration Effects in Machinery, Fluid and Combined Media: Development of a Common Research Approach, New Results

I.I.Blekhman[1 2], L.I. Blekhman [1,2], L.A. Vaisberg [2], V.B. Vasilkov [1,2], K.S. Ivanov [1,2], K.S.Yakimova [1,2]
iliya.i.blekhman@gmail.com

Abstract

New vibration machines and technologies are based on the peculiar effects occurring at high speed impacts in non-linear mechanical systems. Vibrational mechanics and one of its branches, vibrational rheology, represent the general approach to the study of these effects. The report provides an overview of studies covering this type of effects and details the recently discovered new effects and results, which include the phenomenon of vibrational diffusion segregation of granular materials, specific behavior of oscillating objects near the interface of two media and increased buoyancy, suspension of particles in near-wall turbulent flows. The new theoretical developments include the expansion of the range of applicability of the vibrational mechanics approach and the method of direct separation of motions; the application of this method to studies of vibrational effects on any dynamic systems, in particular in the field of physics, chemistry and biophysics; and the development of new screening models.

1 Introduction

Vibration acting on nonlinear systems induces certain motion that represents a superposition of rapid oscillations on a slow motion. This slow motion is usually of the main interest and may be described by equations that differ significantly from the original equations of mechanics by the presence of additional forces, which, according to P.L. Kapitsa, are called vibrational forces [1-3]. These forces are the ones inducing the effects that often seem paradoxical, such as the emergence and disappearance of equilibrium positions of systems, changes in stability characteristics of equilibrium positions and motions, changes in rheological properties of materials in relation to slow or static effects, and the apparent changes in the magnitude and direction of the force of gravity. Non-conservative systems "on the average" tend to become potential and "non-smooth" systems acquire a certain "smoothness".

[1]IPME RAS, Russia
[2]Mekhanobr-Tekhnika Corp., Russia

2 Main Areas of Research

The vibrational mechanics approach has been applied to the following classes of problems [1-6]: 1. The effects of vibration on machinery and machine parts (pendulum and pendulum systems), vibration-induced rotation and termination of rotation under vibration, synchronization of rotors; 2. The effects of vibration on industrial processes; 3. Vibrorheology, the effects of vibration on granular materials, fluids, gas-fluid systems, suspensions, dry friction systems; 4. Problems related to the creation of dynamic vibration materials. These studies laid the basis for the development of a number of new vibration processes and vibrational machines.

3 New Results

1) The expanded application of the direct separation of motions in vibrational mechanics [7]. The main results in this field include application of the method for solving systems of equations that do not satisfy the conditions of theorems N.N. Bogolyubov, V.M. Volosov and B.I. Morgunov, and the use of the perturbation method to systems without small parameters.

2) The generalized application of the vibrational mechanics approach to the problems of vibrational effects on any nonlinear dynamic systems *(oscillatory strobodynamics).* Finding of solutions for a number of problems of such effects on physical, chemical and biological systems [3].

3) The discovery and research of the effect of vibrational diffusion segregation of granular materials [8]. The effect implies that in a granular medium consisting of particles of different sizes, at a sufficiently intense vibration $A\omega^2/g > 5$, where A is the amplitude, ω is the vibration frequency, g is the acceleration of gravity), particles of separate fractions move in the direction opposite to the gradient of concentration of these fractions. In other words, the concentrations of particle fractions tend to equalize, subject to the system of equations similar to nonlinear diffusion equations. This may be illustrated by the following experiment (Fig. 1).

A cylindrical container with circular holes along its side surface of 8 mm in diameter is filled with a mixture of grains of peas ($d_1 \approx 6$ mm) and hazelnuts ($d_2 \approx 15$ mm) in proportion of 1:2 by weight. The vessel was subjected to vertical vibration with amplitude A = 2.2 mm and frequency $\omega = 220$ s^{-1} (35 Hz), which corresponds to $A\omega^2/g = 10.8$. During the first few seconds, over 90% of the peas were leaving the vessel, ejected from the holes; after 60 seconds all peas were almost completely gone. This result is explained by the above-mentioned effect of diffusion of peas intensely moving towards the vessel walls where its concentration is less due to its screening through the holes.

As a model, describing the process of diffusion segregation, its proposed to use theory of bulk material separation under action of vibration, described in [9] (also [1, 3]). According to the aforementioned theory, separation process is described by a system of nonlinear partial differential equations. In a particular case relevant to a current experiment, this system of equations can be simplified into a diffusion equation [10]

Figure 1: Photograph of the experimental setup designed to study the classification of granular materials.

$$\frac{\partial c}{\partial t} = D\Delta c + V\frac{\partial c}{\partial y}, \tag{1}$$

where $c = c(t, r, y)$ is the concentration of particles of the fine fraction; Δ is the Laplace operator in cylindrical coordinates of r, y; V is the absolute of the "slow" vertical velocity of particles of the fine fraction due to gravity ("deterministic" parameter). The initial and boundary conditions are written down as

$$c(0, r, y) = c_0 = \mathrm{const}; \quad \left[\frac{\partial c}{\partial r} + k_R c\right]_{r=R} = 0; \quad \left[\frac{\partial c}{\partial y} - k_h c\right]_{y=0} = 0; \quad \left.\frac{\partial c}{\partial y}\right|_{y=h} = 0 \tag{2}$$

the concentration $c = c(t, 0, y)$ is assumed to be finite. The value R and h corresponds to the vessel radius and to the media layer thickness in respective order, and $k_R > 0$ and $k_h > 0$ are factors describing the permeability of the screening surfaces (which is assumed to be proportional to concentration c near the wall). In contrast to the model shown in Fig. 1, screening is assumed to additionally occur through the bottom of the vessel.

The solution to problem (1), (2) represents a double series. Downward rate of the average concentration of the fine fraction in the vessel $C(t) = \frac{2}{R^2 h}\int\limits_0^h dy \int\limits_0^R c(t, r, y) r dr$ over time is depending on the first member of current series. As a result, the following approximate expression is obtained for $C = C(t)$

$$C \approx c_0 \exp\left(-\left[\frac{\rho_1^2}{R^2} + \frac{s_1^2}{h^2} + \left(\frac{V}{2D}\right)^2\right] Dt\right), \tag{3}$$

where ρ_1 and s_1 are the first roots of transcendental equations

$$\rho J_1(\rho) = k_R R J_0(\rho), \quad \mathrm{ctg} s = \frac{s}{k_h h} + \frac{1}{s}\frac{k_h h + \nu}{k_h h}\nu, \quad (\nu = Vh/2D), \tag{4}$$

J_0 and J_1 are zero- and first-order Bessel functions of the first kind.

When the separation process occurs in a rectangular tray with a width of b , along which a granular mixture is conveyed under vibration, the respective formula takes the following form

$$C \approx c_0 \exp\left(-\left[\frac{q_1^2}{b^2} + \frac{s_1^2}{h^2} + \left(\frac{V}{2D}\right)^2\right] Dt\right), \tag{5}$$

where q_1 is the first root of equation $2\mathrm{ctg}\, q = \frac{q}{k_b b} - \frac{k_b b}{q}$, and k_b is a factor similar to k_R in conditions (2). It may then be very roughly specified that at $k_b b > 2$, the root is $q_1 \approx \pi/2$; at $k_h h > 2$ and $\nu << 1$, $s_1 \approx 4$; and assuming $0.5 < k_R R < 4$, root ρ_1 approximately lies within boundaries $1 < \rho_1 < 2$.

Exponential changes in concentration C were observed in several experiments, which corresponds with the result of aforementioned experiment.

The results obtained have already been used to create highly efficient vibrating separators [11].

4) Mathematical models were suggested for abnormal segregation when under the influence of vibration the process of separation of the components of a granular mixture occurs in the direction of increasing the potential energy of the system (the wedge effect, the Brazil nut effect [3, 8]). These allowed to explain and describe such phenomena as floating boulders in sandy soil under the influence of seismic vibrations, abnormal occurrence of nodules, and buckling of pipeline sections near the sea bottom [8, 12], Fig. 2.

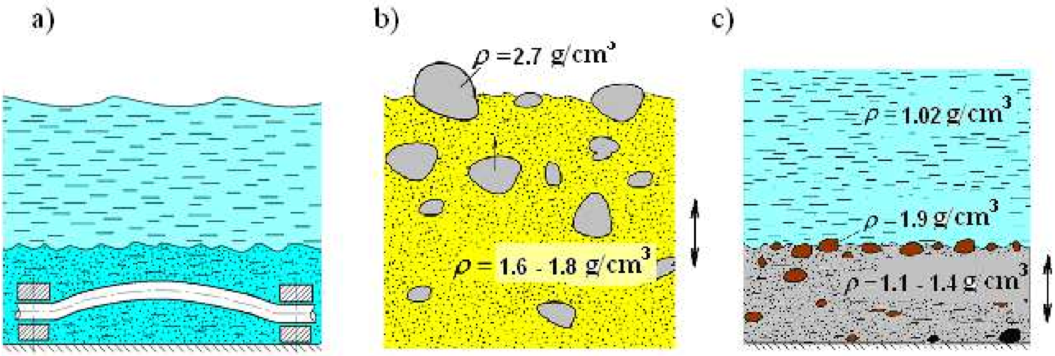

Figure 2: Paradoxical effects under vibration: a) buckling of a pipeline span near the sea bottom, b) buoyancy of boulders in the ground, c) peculiar occurrence of nodules.

5) The behavior of the oscillating bodies near the boundary of two media [12, 13] was studied. As a result, an explanation and mathematical description were obtained for the effect of suspension of solids in near-wall turbulent flows (hydraulic transportation, Fig. 3a), as well as for the effect of increased buoyancy of oscillating bodies. For example, a body with a trapezoidal cross-section (Fig. 3b) rises above the equilibrium level in the absence of oscillations by the value of

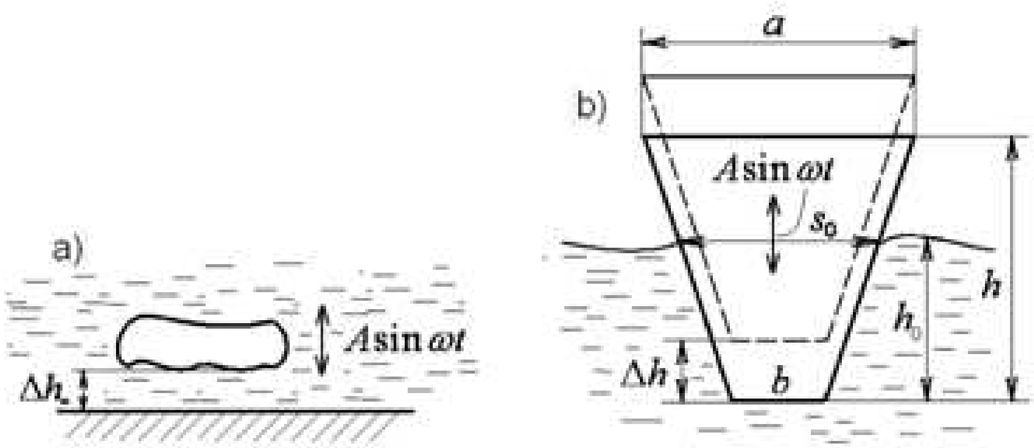

Figure 3: Suspension of vibrating bodies in a fluid: a) suspension of a particle in the bottom fluid flow; b) increased buoyancy of a body with a trapezoidal cross-section.

$$\Delta h = \frac{1}{2}\eta\frac{A^2}{b + 2h_0\eta}, \qquad \eta = \frac{a-b}{2h} \tag{6}$$

where A is the oscillation amplitude.

Let's derivate this simple formula. With the absence of oscillation motion or waves ship immersion depth h_{00} is calculated from the following equation

$$P = Q_{00} = \rho g V_{00} = \rho g[h_{00}b + h_{00}^2\eta]l, \tag{7}$$

that expresses the condition of equalization of the ships weight P by Archimedes force $Q_{00} = \rho g V_{00}$ (l-ship length, Q_{00} - immersed part volume, ρ - liquid density, g - gravity acceleration coefficient).

Let's assume that immersion depth h_0 changes harmonically:

$$h_0 = h_{00} + A\sin\omega t \tag{8}$$

Also we could assume that this change is "fast", i.e. frequency ω are considerably bigger than the frequency of ships small free vertical oscillations. In that case, equation for "current" buoyancy force would be

$$Q_v = \rho g l[(h_{00} + A\sin\omega t)b + (h_{00} + A\sin\omega t)^2\eta, \tag{9}$$

and average for one period

$$\langle Q_v\rangle = \rho g l(h_{00}b + h_{00}^2\eta) + \frac{1}{2}\rho g l\eta A^2 h_{00} = Q_{00} + \Delta Q_0, \tag{10}$$

where $\Delta Q_0 = \frac{1}{2}\rho g l\eta A^2$- additional average buoyancy force.

Presence of this force leads to a decrease of the ships immersed volume (up to the terms of first order with respect to Δh).

$$\Delta V_0 = \rho g s_0 l\Delta h, \tag{11}$$

where

$$s_0 l = \left(b + 2h_{00}\frac{a-b}{2h}\right) l = (b + 2h_{00}\eta) l \qquad (12)$$

- the area of the longitudinal section of the ship. Knowing that $\Delta Q = \Delta V_0 \rho g$, we can derive formula (6). This formula can also be presented in dimensionless form

$$\frac{\Delta h}{A} = \frac{1}{2}\eta \frac{1}{b/A + 2\eta h_0/A}. \qquad (13)$$

If, for example , $\eta = (a-b)/2h = 0.25, \quad b/A = 1, \quad h_0/A = 1$, then

$$\frac{\Delta h}{A} = \frac{1}{2} 0.25 \frac{1}{1 + 2 \cdot 0.25} \approx 0.1,$$

i.e. additional lift of the ship on the wave would be about $1/10^{th}$ of the wave amplitude A. Its noteworthy that the described effect disappears if cross-section of the ship is a rectangle ($a = b, \ \eta = 0, \ \Delta h = 0$).

Also it should be noted that the buoyancy of the body could increase more if we take viscous force into account. In that case extra average buoyancy force is obtained as a result of a natural assumption that resistance of a fluid is lesser when body is moving up than when its moving down [13]. Of course, extensive research of the effect would require solving of a difficult hydrodynamic task. But investigation of a limited case (body oscillations near a solid border) supports this hypothesis [12].

6) New models were developed for the vibratory screening process [10].

7) The effect of vibrational crossing of potential barriers. Detailed studies were conducted for systems in which the barriers are caused by the presence of the gravity force ("vibration against gravity"). These systems have a variety of important technical applications. Two simple basic models of corresponding devices were built.

4 Conclusions

Current paper presents results recently achieved by a group of authors. Main attention is devoted to new results: extension of a general approach to investigation of vibration excitation effects in nonlinear systems; segregation diffusion effect; increase of buoyancy under vibration.

Authors hope that research will be continued by them and other investigators.

Acknowledgements

The studies under subparagraphs 1), 2) and 7) in paragraph 3 were supported by the Russian Science Foundation (grant no. 14-19-01190), the work under subparagraphs 3) and 6) was supported by the Ministry of Education and Science of the Russian Federation (grant no. 14.576.21.0015), and the research under subparagraphs 4) and 5) was supported by the Russian Foundation for Basic Research (grants 14-08-00681 and 13-08-01201 respectively).

References

[1] I.I. Blekhman: Vibrational Mechanics. Nonlinear Dynamic Effects, General Approach, Applications. Singapore et al: World Scientific Pub. Co. Inc, 2000. 509 p.

[2] Selected Topics in Vibrational Mechanics. (Edited by I. Blekhman). Singapore at al.: World Scientific Pub. Co. Inc, 2004. 409 p.

[3] I.I. Blekhman. Theory of Vibration Processes and Devices. Vibrational Mechanics and Vibration Technology. St. Petersburg: Ore and Metals Publishing House, 2013, 640 p. (in Russian).

[4] J.J. Thomsen. Vibrations and Stability. Advanced Theory, Analysis and Tools. 2nd Edition. Berlin, Heidelberg: Springer, 2003. 404 p.

[5] A. Fidlin. Nonlinear Oscillation in Mechanical Engineering. Berlin, Heidelberg: Springer, 2006. 358 p.

[6] V.B. Vasilkov. Influence of Vibration on Nonlinear Effects in Mechanical Systems. Synopsis of Doctor of Engineering. St. Petersburg, 2009. http://www.ipme.ru/ipme/edu/diss/vasilkov.pdf. (in Russian).

[7] V.S. Sorokin. On the Unlimited Gain of a Nonlinear Parametric Amplifer// Mechanics Research Communications, 2014, v. 62, pp. 111-116.

[8] I.I. Blekhman, L.I. Blekhman, L.A. Vaisberg, V.B. Vasilkov, K.S. Yakimova, Vibrational Segregation- Simulation, Experiment and Application to Create new Classifying Machines // Proc. of the XLII Summer School-Conf. APM - 2014. St. Petersburg, June 30-July 5, 2014. St. Petersburg, IPME RAS, 2014, pp. 247-257.

[9] I.I. Blekhman,V.Ya. Khaynman. On a theory of bulk material separation under action of vibration // Engineering Journal (Solid Mechanics). 1968, no. 6, pp. 5-13. (in Russian).

[10] I.I. Blekhman, L.I. Blekhman, L.A. Vaisberg, K.S. Ivanov. Revisiting the Models of Vibration Screening Process // JVE Intern. LTD. Vibroengineering PROCEDIA. 2014, v.3, pp. 169-174.

[11] Patent No. 2550607 RU. Vibrating Screen / L.I. Blekhman, L.A. Vaisberg, V.B. Vasilkov, K.S. Ivanov. Publ. on 10 May 2015, Bulletin No. 13.

[12] L.I. Blekhman. Oscillation-induced Suspension of Solids in Fluid and Loose Media // Bulletin of PNRPU, Mekhanika, Perm: Publishing House of the Perm State Technical University, 2013, no. 2. pp. 52-83. (in Russian).

[13] I.I. Blekhman, L.I. Blekhman, L.A. Vaisberg. Vibration Overcomes Gravity-nonlinear effects, simple models // Proc. of XVIII Intern. Symp. Dynamics of Vibroimpact (Strongly Nonlinear) Systems "DYVIS-2015" (Moscow-Bekasovo, May 17-23, 2015). Moscow: IMASH RAN, 2015. 332 p. (pp. 64-72). (in Russian).

I.I. Blekhman, V.O., Bolshoy pr. 61, St.Petersburg, 199178, Russia