Note 1

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Contents

1 Hot topics: What and why is hot and how to get there

1.1 Materials and structures

The sophistication of the materials reflects the level of technological development of a society. Historically, we call periods by the materials used: Paleo-Neso- or Neolithic, Bronze era, Iron era, etc. The modern history is marked by the invension of steel, aluminum, structured materials, composites, polymers, etc.

Today, we witness an exponential increase of the number of used and designed materials. The used materials are sophisticated: they can change their shape responding to the signal, be magneto-elastic, hyper-plastic, folding, super-deformable, growing, active. The material design involves all scales: from atomic to megascale. It is possible to simulate multiscale picture of the material, control and optimize its properties on all levels.

1.1.1 Composites

Composites possess different (better) properties than any constituents. They have controllable micro-structure. == *Show pictures of negative thermal expansion, Poisson coefficient, effective mass* ==

Problems include:

- optimal structure, bounds on effective properties,
- unusual properties like negative Poisson coefficient and thermal expansion coefficient,
- coupled properties (elastic-magnetic, piezzo-electic)
- percolation and random structures
- high contrast composites and graph of the conducting path
- porosity with multiscale models.

A variety of math methods include: *Homogenization, nonconvex variational problems, rank-one convexity and generalizations, inverse problems, complex variable analysis, probability*

1.1.2 Structural optimization

It can be proven that optimal structures are made from optimal composites. Structural optimization is in demand today because it is possible to make a large variety of structures and it becomes possible to compute and optimize the design.. It become possible to simulate optimization process on the latest generation of computers.

Rocks, minerals, and organic material (wood) are Natural composites. For them, inverse problems are important: determine structural characteristics (volume fraction of the useful mineral) from measurement of effective properties.

Math methods: *Bounds, nonconvex variational problems, numerical optimization, theory of minimax, numerical PDE*

1.1.3 Smart materials and Shape memory alloys

These materials change their shape or physical properties responding to a signal: Example: viscous liquid with metal sand that forms structures responding to magnetic field. Use: Brakes with controllable dissipation.

Shape memory alloys are materials that undergo a phase transition (austenite-martensite). Thin films can change the reflection angle: used in projectors. Ideal switches, actuators.

Math methods: *Nonconvex variational problems, dynamics of unstable materials, numerics*

1.1.4 Resins, wools, elasticity, and thermodynamics

Elasticity of rubber bands, of wool, etc. These materials are different from metals, rocks, or wood, therefore a theory is also different from the classical elasticity. Thermodynamics determines the elastic energy. Open problems:
- control of the properties.

Math methods:*Thermodynamics, Nonlinear elasticity, modeling of complex materials, numerics?*

1.1.5 Flexible, folding materials

Polymers can sustain large deformation. Flexible structures replace an assembly for metal plated, springs, nuts and bolts, etc. The entire mechanism can be produced in one piece. Examples: belt joint, bike gear switch. Mechanics and optimization of such structures is wildly open.

Math methods:*Nonlinear elasticity, multi-stable configurations, large deformations, numerics?*

Thin films A special attention is paid to thin films, their folding, stability, modes of deformation. Two-scale energy: low energy of bending and high energy of plane deformation. Large bending deformation. "Thin film deposition is a crucial step in the manufacture of integrated circuits, semiconductor lasers, disk drives, micro electro mechanical systems (MEMs), solar cells, superconducting devices, turbine blades, and many, many other products. ...

Research in thin film deposition is interdisciplinary, drawing heavily on the thermal and fluid sciences (reacting flows, heat transfer, thermodynamics), and materials science, but increasingly involving control too" (from Caltech website)

Math methods: *Large deformations, buckling, folding, asymptotic of two-scale energy, differential geometry*

1.1.6 Biomaterials

The growing tissues, active materials, bone remodeling, healing, optimality of morphology, soft tissues mechanics.

Example: Why the materials had evolves the way they had? a spiral tree)

Math methods:*Continuum mechanics, modeling of complex materials, optimal design, inverse optimal design, numerics?*

1.1.7 Nanomaterials

Nanomaterial boom is created by the new technology. The most intensive developing area of material science, it required new approaches to design microstructures like nanotubes and nanocoils, their assembly, requires accounting for chemistry and quantum effects. A lot of open areas, but also a very crowded field.

1.1.8 Protein structures

Puzzles with classification, variety, controllability. Math methods ??.

What will be covered

I will discuss nonconvex (multiwell) variational problems that describe "optimal mixtures" and their structures.

1.2 Dynamics

1.2.1 Dynamic of phase transition

Description of the phase transition process, its speed, forming of domen structures.

Shock waves, earthquake motion, and combustion

Propagation of damage, cracks Model of damage. Controllable damage. Damage as protective factor. Waves of damage.

Math. methods: *Nonlinear waves, active materials, finite-dimensional models, simulation.*

1.2.2 Granular materials flow

Packing, nonuniqueness, randomness, energy barrier and instabilities.

1.2.3 Molecular dynamics

1.2.4 Phase transition: Ginzburg-Landau equation

What will be covered

Waves of transition, dynamic homogenization.

1.3 How to find your topic, learn the state of the art, contribute, and disseminate results

1.3.1 Searching for conferences, Universities, Grant agencies, journals, books

1.3.2 Discussion and research groups

It is very difficult if not impossible to adapt all the novel development alone. Therefore, a seminar is needed of the type of book club. In this seminar, participants talk about the novel subjects, researching the sources. A significant progress is expected in a year or two.